Attitude Algorithm Utilised in Mobile Geophysical Measuring Systems

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1 Introduction

Accurate real-time tracking of orientation or attitude of rigid bodies has wide applications in robotics, aerospace, on-land and underwater vehicles, automotive industry, and virtual reality. A quite recent application in geophysics is the analysis of the attitude effect of geophysical sensors such as coil arrays or magnetometers attached to mobile measuring platforms e.g. helicopters and fixed wing airplanes and the correction with respect to the Earth's reference coordinate frame (*Reid etal, 2003, Yin and Fraser, 2004*). Another field of application is wireline logging with a borehole tool. While measuring, the tool spins about its vertical body axis due to the torsion force of the logging cable, which results in a disorientation of geophysical sensors.

In the borehole industry, it is essential to accurately monitor and guide the direction of the drill bit. It is also necessary for an oil rig to log the location of its boreholes at a regular frequency such that the oil rig can be properly monitored. To determine the location of a drill bit in a borehole, it is necessary to know the position and the attitude, which includes the vertical orientation and the North direction.

In prior art systems, a magnetometer is used to determine the magnetic field direction from which the direction of North is approximated. Triple component magnetometers were widely used to sense the tool rotation during a log run. However orientation via magnetometers is subject to restrictions. The employing of magnetometers in boreholes drilled into strongly magnetized formations such as oceanic crust, volcanoes and ore deposits, make it very difficult to utilize the Earth's magnetic field as a reference (Steveling et al, 2003). First, these systems must make corrections for magnetic interference and use of magnetic materials for the drill pipe. Second, systems that rely only on magnetometers to determine North can suffer accuracy degradation due to the Earth's magnetic field variations. Third, a magnetometer alone cannot give an unequivocal measurement of a set of sensor attitude. Measurements made with such a sensor define the angle between the Earth's magnetic field and a particular axis of the sensor. However, this axis can lie anywhere on the surface of a cone of semi-angle equal to that angle about the magnetic vector. Hence, an additional measurement is required to determine attitude with respect to another fixed reference frame.

In order to determine the direction of a spinning borehole tool another type of sensor technology must be utilised to provide orientation that is independent from the Earth's magnetic field as a reference (Galliot et al, 2004, Stoll and Leven, 2002).

The problem to be solved consists of a way to sense the attitude of a borehole tool, which are free to rotate about any direction. A good compromise between accuracy and sensor size are the fibre optical gyros. These devices measure the turning rate about the sensor axis in the tool frame. In this paper an algorithm is presented that approaches the attitude determination based on this discrete data.

2 Principle of Fibre Optical Gyro (FOG)

Gyroscopes are used in various applications to sense the angular rate of turn about some defined axis. The most basic and the original form of gyroscopes make use of the inertial properties of a wheel, or rotor, spinning at high speed. The wheel tends to maintain the direction of its spinning axis in space due to conservation of the angular momentum vector. These devices are susceptible to damage from shock and vibration, exhibit cross-axis acceleration sensitivity and, for the lower cost versions, have reliability problems.

Another type is the vibratory gyroscope. The basic principle of operation of such sensors is that the vibratory motion of part of the instrument creates an oscillatory linear velocity. If the sensor is rotated about an axis orthogonal to this velocity, a Coriolis acceleration is induced. The acceleration modifies the motion of the vibrating element which is an indication of the magnitude of the applied rotation. However, this type of sensor tends to produce biases in the region of 1°/s.

Optical gyroscopes use an interferometer to sense angular motion. An optical gyroscope, laser or fibre, measures the interference pattern generated by two light beams, traveling in opposite directions within a mirrored ring or fibre loop, in order to detect very small changes in motion. This type of gyroscope can be subdivided into fibre optical gyro (FOG), ring laser gyro (RLG), and ring resonator gyro (PARR).

In order to keep track on the orientation of the geophysical sensors the utilisation of fibre optical gyroscopes are now common in navigation and replace prior art systems like mechanical gyros. Important attributes of this new technology are no moving parts, high reliability, stable performance and low costs. Many of its components are based on proven technology from the fiber optical telecommunications industry. Using optical gyros, inertial navigation is accomplished by integrating the output of a set of angular rate sensors to compute the attitude.

In **figure 1** a FOG manufactured by LITEF is shown exemplary. The gyro operates at an optical wavelength of 820 nanometers, with a 110 meter coil of elliptical-core polarization maintaining fiber. The low coherence reduces unwanted interference between waves reflected from the fusion splices used to join the components.

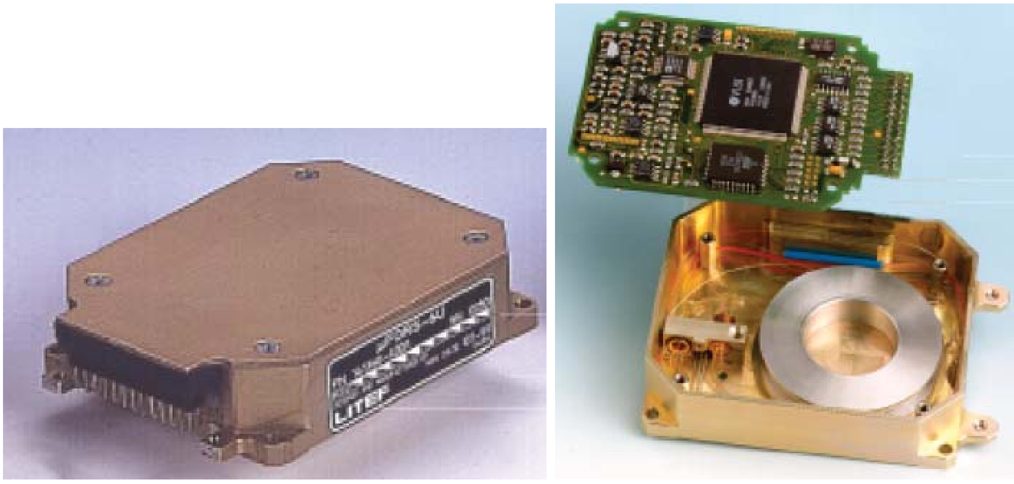

Fig. 1: Example of the μFORS -family (LITEF, Freiburg) (here μFORS-6U)

3 Attitude Computation

Inertial orientation tracking of borehole tools is based upon the same methods and algorithm as those used for aircrafts, ships, and missiles. There is a large quantity of technical literature that describes the fundamentals of inertial navigation technology in great detail, e.g., Grewal et al. (2001), v. Hinüber (1993), Savage (1998a, b), Stovall (1997). If angular rates of a tool are measured constantly with depth, the orientation of the sensors of the tool with respect to inertial space can be determined by applying a suitable coordinate transformation.

A transformation from one coordinate frame to another can be carried out as three successive rotations about different axes. The transformation matrix is given by the product of these three separate transformations as follows:

$$C_b^n = C_3 \cdot C_2 \cdot C_1 \qquad (1)$$

Various mathematical representations can be used to define the attitude of a body with respect to a coordinate reference frame. One of the most common ways of parameterizing the transformation matrix is by use of direction cosine matrix (DCM). Other methods of describing rotations are the use of Euler Angles and Quaternions.

The DCM is a 3x3 matrix, the columns of which represent unit vectors in body axes projected along the reference axes. Matrix C_b^n describes the transformation from coordinate frame “b” to the frame “n” and is written here in component form as follows:

$$C_b^n = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \qquad (2)$$

The element in the i^{th} row and the j^{th} column represent the cosine of the angle between the i^{th} axis of the reference frame "n" and the j^{th} axis of the body frame "b".

When designing a navigation system it is necessary to relate the information from the sensors to a navigation coordinate. The coordinate choice for borehole applications is a geographic or navigation frame "n" with axes {N, E, D}, (North, East and Down). The inertial measurement unit is mounted on the borehole tool constituting a new frame. This is called the body frame "b", and has axes {X, Y, Z}. This frame will be in rotation with respect to the geographic frame. The velocity of this rotation is measured by three orthogonal gyros. The transformation matrix that relates "b" and "n" coordinate frames propagates with time in accordance with the following equation:

$$\dot{C}_b^n = C_b^n\,\Omega_{nb}^b \text{, where } \Omega_{nb}^b = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix} \qquad (3)$$

Ω_{nb}^b is the skew symmetric matrix formed from the elements of the vector $\omega_b^n = \left[\omega_x, \omega_y, \omega_z\right]^T$, which represents the turn rate of the body between the i^{th}-frame and the $(i+1)^{th}$-frame as measured by the gyroscopes in the body frame. In real time applications the integration is implemented with the following approximation

$$C_b^n(t+\delta t) = C_b^n(t)\cdot A(t) \qquad (4)$$

where $\mathrm{A}(\mathrm{t})$ is the DCM which relates the b-frame at time t to the b-frame at time $\mathrm{t}+\delta\mathrm{t}$. For small angle rotations, $\mathrm{A}(\mathrm{t})$ may be written as follows:

$$A(t) = [I + \delta\varphi] \qquad (5)$$

where I is the 3×3 identity matrix and

$$\delta\varphi = \begin{pmatrix} 0 & -\delta\gamma & \delta\beta \\ \delta\gamma & 0 & -\delta\alpha \\ -\delta\beta & \delta\alpha & 0 \end{pmatrix} \qquad (6)$$

in which $\delta\alpha$, $\delta\beta$ and $\delta\gamma$ are small rotation angles through which the body-frame has rotated over the time interval δt about its yaw, pitch and roll axes respectively. If the limit δt approaches zero, small angle approximations are valid and the order of the rotation becomes unimportant.

Hence, the minimum sampling time of the gyros is important for obtaining the transformation matrix with reasonable accuracy. This interval will be a function of the severity of manoeuvres expected from the borehole tool. In many applications the rotation velocity expected is usually less than 25 degrees/sec. With a sampling time of 100 Hz the maximum angle variation will be less than 0.25 degree satisfying the small angle approximation. Rotations with faster dynamics will require smaller sampling time to compute the transformation matrix appropriately.

In order to update the DCM C_b^n, it is necessary to solve a matrix differential equation (3). Over a single computer cycle, from time t_k to t_{k+1}, the solution of the equation may be written as:

$$C_{k+1} = C_k \cdot \exp\int_{t_k}^{t_{k+1}} \Omega\, dt \qquad (7)$$

Provided that the orientation of the turn rate vector ω remains fixed in space over the update interval, we may define:

$$\int_{t_k}^{t_{k+1}} \Omega \, dt = [\sigma \times] \qquad (8)$$

and

$$C_{k+1} = C_k \cdot \exp[\sigma \times] = C_k \cdot A_k \qquad (9)$$

Where C_k represents the direction cosine matrix which relates body to reference axes at the k^{th} computer cycle, and A_k the direction cosine matrix which transforms a vector from body co-ordinates at the k^{th} computer cycle to body co-ordinates at the $(k+1)^{th}$ computer cycle.

The variable σ is an angle vector with direction and magnitude such that a rotation of the body frame about σ through an angle equal to the magnitude of σ will rotate the body frame from its orientation at computer cycle k to its position at the computer cycle k+1. The components of σ are denoted by σ_x, σ_y, and σ_z and its magnitude given by:

$$\sigma = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} \quad (10)$$

and

$$\sigma \times = \begin{pmatrix} 0 & -\sigma_z & \sigma_y \\ \sigma_z & 0 & -\sigma_x \\ -\sigma_y & \sigma_x & 0 \end{pmatrix} \quad (11)$$

Expanding the matrix exponential function in a power series gives:

$$A_k = I + [\sigma \times] + \frac{[\sigma \times]^2}{2!} + \frac{[\sigma \times]^3}{3!} + \frac{[\sigma \times]^4}{4!} + \dots \qquad (12)$$

where

$$[\sigma \times]^2 = \begin{pmatrix} -(\sigma_y^2 + \sigma_z^2) & \sigma_x \sigma_y & \sigma_x \sigma_z \\ \sigma_x \sigma_y & -(\sigma_x^2 + \sigma_z^2) & \sigma_y \sigma_z \\ \sigma_x \sigma_z & \sigma_y \sigma_z & -(\sigma_x^2 + \sigma_y^2) \end{pmatrix} \qquad (13)$$

$$[\sigma \times]^3 = -\sigma^2 \cdot [\sigma \times] \quad (14)$$

$$[\sigma \times]^4 = -\sigma^2 \cdot [\sigma \times]^2 \quad (15)$$

Thus we may write

$$\begin{aligned} A_k &= I + [\sigma \times] + \frac{[\sigma \times]^2}{2!} - \sigma^2 \frac{[\sigma \times]}{3!} - \sigma^2 \frac{[\sigma \times]^2}{4!} + \dots \\ &= I + \left(1 - \frac{\sigma^2}{3!} + \frac{\sigma^4}{5!} - \dots\right) \cdot [\sigma \times] + \left(\frac{1}{2!} - \frac{\sigma^2}{4!} + \frac{\sigma^4}{6!} - \dots\right) \cdot [\sigma \times]^2 \end{aligned} \qquad (16)$$

and which may be written as follows:

$$A_k = I + \frac{\sin \sigma}{\sigma} [\sigma \times] + \frac{(1 - \cos \sigma)}{\sigma^2} [\sigma \times]^2 \qquad (17)$$

Equation (17) is input in equation (9) that updates the frame at time t to the frame at time t+Δt. To compute the attitude of the tool, first the measured rotation rates are input in equation 17. Then, beginning with $C_0 = I$ the rotation matrices C_{k+1} for each computer cycle *k*+1 are successively computed by equation (9). The tool attitude $\vec{r}_k^{\,n}$ in the external frame at the k^{th} cycle results to

$$\vec{r}_k^{\,n} = C_k \cdot \vec{r}_0^{\,n} \qquad (18)$$

with the starting orientation $\vec{r}_0^{\,n}$.

4 Attitude Computation allied to the Göttingen Borehole Magnetometer (GBM)

The Göttingen Borehole Magnetometer (GBM) contains a triple of μFORS-36M sensors, which are attached rigidly to the borehole tool. The rotation axes of the sensors are aligned with the body axes of the tool, and thereby sense the rotation rate about the vertical axis and the tilting movement about the two horizontal axes. Angular rates are output with a sample rate of 1 second. The salient specifications of the μFORS-36m are the maximum input rotation rate of ±720°/s that allows sensing a maximum rotation rate of 2 revolutions per second with a resolution of $9 \cdot 10^{-5}$ °. Moreover, the dimensions of the sensor (53mm x 58mm x 19mm) are small enough to fit to the inner diameter of 65mm of the GBM. This is important to reduce the overall diameter of the tool and make it applicable in narrower boreholes. The bias drift is $<$6°/h (1σ) and is reduced to <3°/h at working temperature of 30 °C to 50 °C. The maximum operating temperature range is -40 °C to +75 °C.

To check the performance of the attitude computation, the GBM was brought to the Gauss Haus at the Geophysical Institute of the University of Göttingen where it was fixed to the ceiling and hung vertically down. Then it was exposed to different kinds of movements like tilting and rotation about its vertical axis or combined movements. This test is visualized in two video clips, video 1 and video 2. Here, the tool attitude with the corresponding turning rate and the magnetic field data are shown in three components. The maximum rotation rate was 150°/s. In this case the rotation angles are no longer small, which resulted in an erroneous representation of the attitude of the tool. **Figure 2** displays the movement of the GBM with respect to the {N, E, D}, (North, East and Down) coordinate frame

Figure 2

Acknowledgments

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