#### Motionally Induced Electromagnetic Field within the Ocean

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#### Abstract

The contribution of motionally induced electromagnetic (EM) fields at the seafloor is generally considered small, but since the characteristic reservoir signal in marine controlled source electromagnetic (mCSEM) data is also small, the inclusion of the motional induction contribution in modelling the signal will enhance the probability of reservoir detection. Here, we have studied the electromagnetic induction caused by ocean water flow with in earth's magnetic field.

When a charge particle moves with certain velocity in earth's magnetic field, it experiences a Lorentz force. The action of Lorentz force generates a secondary electric field through galvanic and inductive processes. For the mathematical formulation, we considered Lorentz electric field as a source in the corresponding set of Maxwell's equations. We solved these Maxwell's equations for a 1D model and velocity structure using two different Green's function i.e. a two half space Green's function and a layered Green's function. The layered Green's function is especially useful in studying the sensitivity of electric and magnetic field for different conductivity structures in the earth. Further, the signal variation with the conductivity of ocean, depth of ocean and wave velocity is studied to profoundly understand the effect of these parameters.

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# **1. Introduction**

Oceanic water movements in the earth's magnetic field induce electric and magnetic fields in the ocean. The induced fields are signal for oceanographic and seismological applications but noise for magnetotelluric (MT) and marine controlled source electromagnetic (mCSEM) applications. Oceanographers and seismologists use the fields respectively to study the velocity structure and the seismic background noise. The MT and mCSEM signals, which are generally used for lithospheric and hydrocarbon exploration studies, are contaminated by the induced field and act as noise. Our prime focus of motional induction study here is on mCSEM problems; nevertheless the results are also applicable for other scientific applications.

Generally, motionally induced noise in seafloor mCSEM data is considered small, but since the characteristic reservoir signal is also small, the understanding and possible removal of noise may be essential to increase the number of possible target reservoirs. The electromagnetic induction investigation has a long history. Initially, by an observation of deflection in the galvanometer by stream waves Faraday (1832) concluded that flow of water will induce electric currents, which was afterwards measured experimentally by Young et al (1920). Later it was almost neglected and got re-attention after World War II. Longuet-Higgins et al., (1954) initiated the investigation of electric field induction by surface waves. Crews and Futterman (1962) investigated the magnetic field induction by oceanic movement. Further, Sanford, (1971) extended the theory of motional induction by considering three-dimensional water flows. A comprehensive study of the theory has been made by Podney (1975), who generalised and extended the previous work by removing the restrictive assumptions imposed on ocean-water velocity field. Chave (1983) generalised the EM induction process by considering driving electric field term (i.e.  $\vec{v} \times \vec{B}$ ) and further, Chave and Luther (1990) re-examined the motional induction problem.

A primary purpose of this paper is to present a generalised simple illustrative theory for the problems of motional induction. For the purpose, we have formulated a set of Maxwell's equation for our problem. These equations are simplified for electric and magnetic field components by considering a horizontally progressing ocean wave and then solved by using the Green's function, with appropriate boundary conditions. Here, we solved the problem with two different Green's functions. For a uniformly conductive earth, a two half space Green's function is utilised and for a layered earth, a layered Green's function is utilised. A two half space Green's function doesn't offer reflections because of homogeneous conductivity consideration and thus expresses only the case of downward progressive diffusive waves. The layered Green's function includes both downward and upward propagating diffusive waves offered by the layered boundaries.

#### 2. Problem formulation in terms of Maxwell's equation

# 2.1 Basics

An electrically conducting fluid like ocean consists of charged particles. The particles in the ambient geomagnetic field experience a deflective Lorentz force. If ' $\vec{v}$ ' is the velocity of charge particle 'q' moving in the geomagnetic field ' $\vec{B}_0$ ', then the Lorentz force is:

$$\vec{F}_{L} = q(\vec{v} \times \vec{B}_{0}) \tag{1}$$

The charge 'q' experiences the deflecting force ' $\vec{F}_L$ ' because of the action of an electric field which we call as Lorentz electric field ( $\vec{E}_L$ ),

$$\vec{F}_{L} = q\vec{E}_{L} \tag{1-a}$$

Therefore,

$$\vec{E}_{L} = \vec{v} \times \vec{B}_{0} \tag{2}$$

The field ' $\vec{E}_L$ ' generates a secondary electric field ' $\vec{E}$ ', mainly by two processes-

- (1) **Galvanic process:** At locations, where  ${}^{'}\bar{E}_{L}{}^{'}$  has a component parallel to the conductivity gradient  ${}^{'}\nabla\sigma'$  ( $\sigma$  is conductivity in S/m), space/surface charges are accumulated. The accumulated charges galvanically create a secondary field  ${}^{'}\bar{E}{}^{'}$ , even if the wave velocity is constant.
- (2) **Inductive process:** The magnetic field ' $\vec{H}_L$ ' is created by the current density ' $\vec{J}_L$ ' i.e.

$$\vec{J}_{L} = \sigma(\vec{v} \times \vec{B}_{0})$$
 
$$\nabla \times \vec{H}_{L} = \vec{J}_{L}$$

when the velocity '  $\vec{v}$  ' changes with time. This induces a secondary electric field  $\vec{E}$  via the law of induction.

We assume that all the hydrodynamics, including Coriolis force, is included in the given ' $\overline{v}$ '. We do not care about the sources of forces. Finally, for a stationary frame of reference, the current density in the Ohms law is given by

$$\vec{J} = \sigma(\vec{E} + \vec{E}_L) = \sigma(\vec{E} + \vec{v} \times \vec{B}_0)$$
(3)

Here, ' $\sigma$ ' is the conductivity of the fluid, ' $\sigma \vec{E}$ ' is the current term generated by both a galvanic and an inductive process and ' $\sigma(\vec{v} \times \vec{B}_0)$ ' is the source current term for motional induction case. Under the quasi-static approximation, the set of Maxwell's equation to be solved for the motional induction case is

$$\nabla \times \vec{H} = \sigma(\vec{E} + \vec{v} \times \vec{B}_0)$$

$$\nabla \times \vec{E} = -\mu_0 \partial_t \vec{H}$$
(4- a, b)

In an exact formulation, the ambient magnetic field is the sum of geomagnetic field ' $\vec{B}_0$ ', the external magnetic field by ionospheric currents ' $\vec{B}_{ext}$ ', the field generated by small local anomalies ' $\vec{B}_{1-ano}$ ' and the motionally induced field ' $\vec{B}_{mi}$ ' i.e.  $\vec{B} = (\vec{B}_0 + \vec{B}_{ext} + \vec{B}_{1-ano} + \vec{B}_{mi})$ . In general, the last three terms are orders of magnitude smaller than the geomagnetic field  $\vec{B}_0$ . Therefore, for computations, the local value of earth's magnetic field would be a good choice.

# 3. A halfspace model

Let the north, east and positive downward directional depth in Cartesian coordinate system be represented by  $\hat{x}$ ,  $\hat{y}$  and  $\hat{z}$ . Consider a two halfspace model with the boundary at the earth surface (i.e. z = 0). Let the insulating air halfspace extend in the negative vertical upward direction while the earth halfspace extends in positive vertical downward direction from the boundary surface (Fig 1). The conductivity of the earth's half space is  $\sigma = 3.33$  S/m, which contains an ocean and sediments below. In general, the conductivity of the ocean depends on dissolved ions content and their mobility, which is primarily a function of temperature and pressure. The conductivity in warm, top shallow water is normally high ( $\approx 5$  S/m) compared to cold, deep water ( $\approx 2.5$  S/m). For simplicity, oceans can be considered homogeneous as the range of conductivity difference (i.e. 2.5–5 S/m) is small. Consider with depth d=1000 m ocean moving with a velocity  $\bar{v}$  in the x-direction (Fig 1). Below the ocean, a subsurface of stationary sediments (i.e.  $\bar{v} = 0$ , motionless) exists.



Fig 1. The two halfspace model. z = 0 is the boundary between earth and air halfspace. Depth is considered positive in the downward direction. The earth halfspace consists of two layers. The top layer represents the ocean having non-zero velocity in the x-direction. The depth of the ocean is d. Below the ocean, a second layer of static (i.e. velocity zero) sediments extends. The conductivity and magnetic permeability of earth halfspace is  $\sigma$  and  $\mu_0$  respectively.

# Let $\vec{B}_0 = B_0 \hat{z}$ and $\vec{v} = v_x(z,t)\hat{x}$

This will generate a horizontal electric field perpendicular to the velocity direction and a horizontal magnetic field perpendicular to the electric field, and thus we can write:

$$\dot{\mathbf{E}} = \mathbf{E}_{\mathbf{y}}(\mathbf{z}, \mathbf{t}) \hat{\mathbf{y}} \text{ and } \ddot{\mathbf{H}} = \mathbf{H}_{\mathbf{x}}(\mathbf{z}, \mathbf{t}) \hat{\mathbf{x}}$$

From eq. (4), we have

$$\partial_{z}^{2} E_{y} = \mu_{0} \sigma \left( \partial_{t} E_{y} - B_{0} \partial_{t} v_{x} \right)$$
(5)

Assume a harmonic time dependence for simplicity,  $v_x(z,t) = \tilde{v}_x(z) e^{+i\omega t}$ ,

Then  $E_y(z,t) = \widetilde{E}_y(z)e^{+i\omega t}$ ,  $H_x(z,t) = \widetilde{H}_x(z)e^{+i\omega t}$  and eq. (5) reduces to

$$\partial_z^2 \widetilde{E}_y - k^2(z) \widetilde{E}_y(z) = -g(z)$$
 (6)

<sup>23.</sup> Schmucker-Weidelt-Kolloquium für Elektromagnetische Tiefenforschung,

where,  $g(z) = +k^2(z) \tilde{v}_x(z) B_0$  is the source term,  $k(z) = \sqrt{i\omega\mu_0\sigma(z)}$  represents the electromagnetic damping phenomenon. It is complex such that amplitude decay is associated with a phase shift with respect to the field at the surface. The Green's function  $G(z \mid z_0)$  corresponding to equation (6) is defined by

$$\partial^2 G(z \mid z_0) - k^2(z)G(z \mid z_0) = -\delta(z - z_0)$$
(7)

Crosswise multiplication with G and  $\widetilde{E}_{v}$  and subtraction yields

$$\partial_{z}(G\partial_{z}\widetilde{E}_{y} - \widetilde{E}_{y}\partial_{z}G) = -g(z)G + \widetilde{E}_{y}\delta(z-z_{0})$$
 (8)

The integration of (8) over depths z yields the solution

$$\widetilde{E}_{\mathbf{y}}(z_0) = \int_0^d g(z) G(z \mid z_0) dz, \text{ for } -\infty < z_0 < \infty$$
(9)

i.e. a convolution of the source term with the Green's function. The integration, which formally must be carried out from minus to plus infinity, reduces to 0 to d, because that is where the source is nonzero. For the horizontal electric field  $\tilde{E}_{y}(z_{0})$  calculation at any desired depth, knowledge of the Green's function is required.

# **3.1 Green's function**

#### 3.1.1. Half space

The Green's function is commonly used to solve inhomogeneous boundary value problems. The solution by means of Green's function gives a special advantage because of its reciprocity property, which states 'relationship between a oscillating source and the resulting field at some point of observation is unchanged even if the observation and source points are interchanged'. Using the boundary conditions, the Green's function is calculated for the halfspace. Calculation is as follows:

For an arbitrary small  $\varepsilon$ , equation (7) follows,

$$\partial_{z} \mathbf{G}(\mathbf{z}_{0} + \varepsilon \mid \mathbf{z}_{0}) - \partial_{z} \mathbf{G}(\mathbf{z}_{0} - \varepsilon \mid \mathbf{z}_{0}) = -1$$
(10)

The second boundary condition is that  $G(z | z_0)$  is continuous at  $z = z_0$  i.e.

$$G(z_0 + \varepsilon | z_0) - G(z_0 - \varepsilon | z_0) = 0$$
(11)

For the uniform halfspace,

$$G(z \mid z_0) = \begin{cases} A e^+ kz & \text{for } z < z_0 \\ B e^- kz & \text{for } z > z_0 \end{cases}$$
(12-a,b)

A and B are determined from (10) and (11) which yields the Green's function for the half space

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$$G(z \mid z_0) = \frac{1}{2k} e^{-k|z-z_0|}$$
(13)

#### **3.1.2 Two halfspaces**

In order to consider the diffusive reflection and transmission effects due to boundaries, let us consider two uniform half spaces with  $k(z) = k_0$  in the air (i.e. z<0) and k(z) = k in earth (i.e. z>0). The Green's function, for  $z_0>0$ , can be written as

$$G(z \mid z_0) = \begin{cases} \frac{1}{2k} e^{-k} \mid z - z_0 \mid + R e^{-kz} & \text{for } z \ge 0\\ T e^{-k_0 z} & \text{for } z \le 0 \end{cases}$$
(14)

where, T and R are respectively transmission and reflection coefficients. The continuity of G and  $\partial_z G$  at z = 0 yields

$$G(z \mid z_{0}) = \begin{cases} \frac{1}{2k} \left[ e^{-k \mid z - z_{0} \mid} + \frac{k - k_{0}}{k + k_{0}} e^{-k(z + z_{0})} \right] & \text{for } z \ge 0, z_{0} \ge 0\\ \frac{1}{k + k_{0}} e^{k_{0}z - kz_{0}} & \text{for } z \le 0, z_{0} \ge 0 \end{cases}$$
(15)

In particular, for the insulating air halfspace  $\mathbf{k}_0=\mathbf{0}$  , the two halfspace Green's functions take the form

$$G(z \mid z_0) = \begin{cases} \frac{1}{2k} \left[ e^{-k \mid z - z_0 \mid} + e^{-k(z + z_0)} \right] & \text{for } z \ge 0, z_0 \ge 0 \\ \frac{1}{k} e^{-kz_0} & \text{for } z \le 0, z_0 \ge 0 \end{cases}$$
(16)

#### 3.1.3. A simple model and field expression

Equation (9) includes the source term g(z), which depends on conductivity and velocity, both are depth dependent. For simplicity, consider a uniform horizontal velocity i.e.  $v(z) = v_0$ ,  $0 \le z \le d$ . Let the depth dependent conductivity be zero in the air and  $\sigma = 3.33$  S/m for the earth i.e.

$$\sigma(z) = \begin{cases} \sigma, z \ge 0\\ 0, z < 0 \end{cases}$$

The equation (9), (16) and (4), yields the horizontal electric and magnetic field expression for the full space.

Electric field in full space:

$$\begin{split} \widetilde{E}_{y}(z_{0}) &= v_{0} B_{0}(1 - e^{-kd}) , z_{0} \leq 0 \\ \widetilde{E}_{y}(z_{0}) &= v_{0} B_{0}(1 - e^{-kd} \cosh(kz_{0})) , 0 \leq z_{0} \leq d \\ \widetilde{E}_{y}(z_{0}) &= v_{0} B_{0}e^{-kz_{0}} \sinh(kd) , z_{0} \geq d \end{split}$$
(17-a, b, c)

Magnetic field in full space:

$$\begin{split} \widetilde{H}_{x}(z_{0}) &= 0 &, z_{0} \leq 0 \\ \widetilde{H}_{x}(z_{0}) &= -\frac{kv_{0} B_{0}}{i\omega\mu_{0}} e^{-kd} \sinh(kz_{0}) &, 0 \leq z_{0} \leq d \\ \widetilde{H}_{x}(z_{0}) &= -\frac{kv_{0} B_{0}}{i\omega\mu_{0}} e^{-kz_{0}} \sinh(kd) &, z_{0} \geq d \end{split}$$
(18-a, b, c)

The graphical response of (17) and (18) is shown in Fig (2). The response is calculated for a halfspace model (Fig 1) of a conductivity of 3.33 S/m containing 1000 m thick layers of ocean and sediments. The magnetic permeability is kept constant for both halfspace and is equal to free space permeability i.e.  $\mu_0 = 4\pi \times 10^{-7}$  Vs/Am. The ocean has a homogeneous velocity of 10 cm/s in an ambient geomagnetic field of  $5 \times 10^{-5}$  T. The responses are studied for frequencies 0.001, 0.01, 0.1, and 1 Hz. In general, the electric field becomes gradually weaker with depth (Fig 2). For 1 Hz, rather than weakening gradually, the electric field amplitude increases and becomes strongest with in the ocean (at 400 m depth). Further, the field  $B_x$  is zero at the surface and progressively becomes stronger with respect to depth. At the ocean bottom, it offers strongest amplitude. The zilch of  $B_x$ at the earth surface is for the reason that in air halfspace field  $\widetilde{E}_{\rm y}$  is constant and therefore  $\partial_z \widetilde{E}_v = 0$  in particular, which is  $B_x$  (eq. 4-b). Further, as far as frequency based variation are concerned, the smallest frequency  $B_x$  produces the strongest amplitude at the ocean bottom. The field  $E_y$  shows a contrary behaviour. Here the smallest frequency offers the weakest amplitude at the ocean floor. At large frequencies the field  $E_y$  can not reach to the ocean floor, because of the shallow skin depth, offers constant amplitude (Fig 3).

The important result in Fig 2 & 3 is that the field  $E_y$  is smooth over the boundary between the ocean and sediment. On the other hand, the field  $B_x$  offers a sharp change in pattern there (at boundary). Note that the conductivity of the ocean and subsurface (sediments) are identical. They differ only in their velocity state (subsurface is static and

ocean is dynamic). Therefore the sensitivity of the field  $B_x$  at the boundary is interpreted as result of velocity change.



Fig 2. Variation of the horizontal comp. electric  $(E_y)$  and magnetic  $(B_x)$  field with respect to depth. Variation is shown for four frequencies viz. 0.001, 0.01, 0.1 and 1 Hz. An ocean of conductivity  $\sigma$ =3.33 S/m extends from surface to 1000 m depth (i.e.  $0 \le z \le 1000$  m). Below the ocean (i.e. below 1000 m depth) there is a sediment layer with the same conductivity as the ocean (i.e.  $\sigma$ =3.33 S/m). The horizontal line at 1000 m is to show the separation of these two boundaries.



Fig 3. Variation of the horizontal component electric  $(E_y)$  and magnetic  $(B_x)$  field with respect to depth. Variation is shown for three frequencies viz. 2, 10 and 50 Hz. Other information is the same as in Fig 2. It is evident that at the ocean floor, the field  $E_y$  offers almost same amplitude for all the three frequencies.  $B_x$  is strongest and weakest respectively for the smallest and largest frequency.

The field  $E_y$  (17-b) with in the ocean (i.e.  $0 \le z_0 \le d$ ) consists of two terms. The first term is the kernel for the motional induction, while the second governs the depth dependent frequency based EM damping. From the expressions (17-b & 18-b), it is clear that the strength of both the fields  $E_y$  and  $B_x$  depends on the ocean depth (i.e. ocean deepening) because of the factor  $e^{-kd}$ . Effect of the ocean deepening is studied for oceans of thickness (depth) varying from 1000 m to 9000 m. Observation depth is constant for all practical cases and is 1000 m (i.e.  $z_0 = 1000$  m). Results are shown in Fig (4).  $E_y$  is strongest for the thickest ocean and weakens down gradually as ocean shallows up. The reverse is observed for the  $B_x$  case with weakest strength in the deepest ocean which progressively becomes stronger as the ocean gradually shallows up. The sensitivity for the deepening effect depends on frequency and therefore on skin depths. For that reason, the deepening effect is more effective at smaller frequencies. The skin depths, for a halfspace of 3.33 S/m, at frequencies 0.001, 0.01, 0.1, 1 and 10 Hz are approximately 8664, 2739, 866, 273 and 86 m, respectively.

The factor which may significantly influence the motional induction is the conductivity of the ocean. The results of the conductivity variation are shown in Fig 5. The field strength is observed at 1000 m depth for conductivities varying from 3 to 5 S/m. Observations are made for five different frequencies (i.e. 0.001, 0.01, 0.1, 1 and 10 Hz). The results suggest that the field  $B_x$  is more sensitive to conductivity variation than the field  $E_y$ . Further, lower frequencies are more responsive to conductivity variation than high ones because of the thicker skin-depth.



**Fig 4. Deepening effect of an ocean:** Conductivity of ocean kept constant (i.e. 3.33 S/m). For 1000 to 9000 m deep ocean the field strengths are calculated at frequencies 0.001, 0.01, 0.1, 1 & 10 Hz. The y- and x-axis represents ocean depths and field strength respectively. The observation depth is constant and is 1000 m. The magnetic component for 1 and 10 Hz is truncated for depths greater than 5000 and 2000 m respectively to save the Figure from masking and cluttering.



**Fig 5. Conductivity variation effect:** The conductivity of the ocean is varied from 3 to 5 S/m and the effect is studies for frequencies 0.001, 0.01, 0.1, 1 & 10 Hz. The horizontal component electric and magnetic field strength, observed at 1000 m ocean depth is shown w.r.t. conductivities for the chosen frequencies. Electric and magnetic components both are sensitive to the conductivity of the ocean. Sensitivity increases with the decreasing frequency.

Equation (17) and (18) indicate that the field  $B_x$  is more sensitive to conductivity variation in vertical direction, as conductivity is effectively convolved, than the field  $E_y$ . But it would not be appropriate to discuss conductivity variation in vertical direction using a halfspace model. Therefore, we will be back on this issue with a justified layered model and a layered Green's function.

Moreover, the equation (17) and (18) implies that the increase in the wave velocity  $v_0$  will cause a constant shift in the  $E_y$  and  $B_x$  field. Three experiments are conducted for wave velocities of 10, 1 and 0.1 cm/s to study the effects. The results are shown in the Fig 5. A log scale is used for x-axis plotting for clarity reasons. As at the

surface  $B_x=0$ , therefore the log scale plot starts from 100 m depth as 100 m is the 2<sup>nd</sup> discretized layer of the ocean. The selection of velocities is such that the 1<sup>st</sup> selection exceeds the 2<sup>nd</sup> by factor of 10 and same is true for 3<sup>rd</sup> and 2<sup>nd</sup>. Evidently, response for the field  $E_y$  and  $B_x$  also exceeds by a static shift of factors of 10. The graph clearly illustrates that the velocity is an important parameter for practical simulation, any bias in velocity leads to the same bias in EM field.



Fig 6. Wave Velocity effect. Semi-log plot of horizontal electric and magnetic field component illustrating the effect of velocity change. The strongest velocity generates the strongest EM field. The Bx plots starts from 100 m depth as the strength of Bx at z=0 is zero and at z=100 m the next layer starts.

Fig 7. Variation of the horizontal component electric ( $E_y$ ) and magnetic ( $B_x$ ) field with respect to depth for a layered model. Variation is shown for four frequencies viz. 0.001, 0.01, 0.1 and 1 Hz. An ocean of conductivity  $\sigma$ =3.33 S/m extends from the surface to 1000 m depth (i.e.  $0 \le z \le 1000$  m). Below the ocean there, is a 1000 m thick sediments layer with conductivity  $\sigma$  =1 S/m. A 100 m thick reservoir of conductivity 0.01 S/m is embedded at 2000 m depth. The horizontal line marks the boundary of different formation.

#### 4. Layered model

# 4.1. Layered Green's function

Let us consider a layered Green's function defined as

$$G(z \mid z_0) = \frac{1}{2k_1} \left[ e^{-k_1 \mid z - z_0 \mid} + R_0 e^{-k_1 z} + R_d e^{+k_1 z} \right]; \quad 0 \le z \le d, \ 0 \le z_0 \le d$$

where,  $k_1 = \sqrt{i\omega\mu_0\sigma_1}$  is the electromagnetic damping in the ocean,  $R_0$  describes the field diffusing downward by reflection from the air-earth interface and  $R_d$  describes the field diffusing upward by reflection from the seafloor (i.e. z = d). In this particular case the constants  $R_0$  and  $R_d$  are determined from boundary conditions:-

a)  $\partial_z G(0 \mid z_0) = 0$ , Since  $G(z \mid z_0)$  is constant in the air halfspace

b)  $\partial_z G(d\,|\,z_0)+\mu_0 b_2(d) G(d\,|\,z_0)=0$  , where,  $b_2(d)$  is determined via continuous transfer function.

They yield,

$$\begin{split} R_{0} &= \frac{1 - R_{c} e^{-2k_{1}(d_{1} - z_{0})}}{1 + R_{c} e^{-2k_{1}d_{1}}} e^{-k_{1}z_{0}} \qquad ; \qquad R_{d} = -\frac{R_{c}(1 + e^{-2k_{1}z_{0}})}{1 + R_{c} e^{-2k_{1}d_{1}}} e^{-k_{1}(2d_{1} - z_{0})} \\ \text{with } R_{c} &= \frac{\mu_{0}b_{2} - k_{1}}{\mu_{0}b_{2} + k_{1}} \end{split}$$

# 4.2 Layered model and Response

EM field responses calculated for a layered model are shown in Fig (6). The parameters of the model are tabled in table 1. The magnetic permeability of the each layer is equal to free space permeability ( $\mu_0$ ). The response is computed for four frequencies viz. 0.001, 0.01, 0.1 and 1 Hz. It is evident that the field  $E_y$  varies smoothly over layer boundaries, indicating its sensitivity to the vertically averaged conductivity rather than conductivity variation at layered boundaries. However, the field  $B_x$  senses each layer and offers a change in field strength at each layer boundary. Evidently, the magnetic field and its sensitivity for conductivity variation is frequency dependent. At a small frequency the field is strong and vice versa.

**Table 1: Layered Model** 

Layers	Thickness (m)	Conductivity (S/m)	Other Parameters
Ocean	1000	3.33	Wave Velocity=0.1 m/s
Sediments	1000	1	Ext. Magnetic Field=5×10 <sup>-5</sup> T
Reservoir	100	0.01	Magnetic Permeability of each layer
Below	1000	1	$=4\pi \times 10^{-7}$

# **5. Discussion and Conclusion**

No doubt, the actual earth is 3D and therefore the electromagnetic response of the earth will always be more complex than the response of any simplified model. However, a simplified model, roughly representing actual earth layers can be used to validate results. The choice of the model dimension (i.e. 1D, 2D or 3D) sometimes has a large impact on the result. In particular, for mCSEM surveys, generally 2D data acquisition methodology is followed and therefore a 2D/3D modelling may lead to a better interpretation. However, 1D model formulations are simple and give good insight into the physics of the problems.

Field strength depends on their components (depending on the physical process involved), whose production mainly depends on the involvement of the source components. For example, a 1D model in motional induction study results in  $E_y$  and  $B_x$  (two) components when a 1D source (i.e. wave-velocity in x-direction) is considered. However, a 2D source (i.e. wave-velocity in x-direction and a wave wavelength in y-direction) consideration results in  $E_y$ ,  $B_x$  and  $E_z$  (three) components. This suggests, even a 1D model, with a proper source formulation offers significant insight to the problems. Further, the simplicity of a 1D model is an important advantage. In this paper we thoroughly looked for the sources of the motional induction in the ocean, which is incorporated in 1D model for theoretical development. The horizontal (x-direction) wave motion and a vertical (z-direction) geomagnetic field consideration lead to the excitation of the field components ' $E_y$ ' and ' $B_x$ '. These fields illustrate some of important results and effects of the ocean dynamics in varying oceanic conditions.

In general, for a uniform halfspace, there is gradual reduction and increase respectively in the strength of  $E_y$  and  $B_x$ , w.r.t. ocean depth (Fig 2). This conclusion is valid for frequencies with skin depth ' $\delta$ ' greater than the ocean depth 'd'. For the case when  $\delta < d$ ,  $E_y$  may show maximum amplitude somewhere in the ocean, rather than at the ocean surface. On the other hand in the ocean the field  $B_x$  always offers maximum strength at the floor and minimum (i.e  $B_x = 0$ ) at the surface. Since  $B_x$  vanishes at the surface, the mCSEM surface measurement may offer significant noise-free signals there, if MT field is avoided. At the ocean floor as  $B_x$  always has maximum strength therefore it is necessary to correct mCSEM data for motionally induced field. It is evident from Fig (2), even in a halfspace where the conductivity of ocean and subsurface is same,  $B_x$  senses the boundary of the ocean and subsurface (i.e. sediments) though  $E_y$  does not see it. Still, an important question left to answer is 'which field is/are sensitive to layer demarcation, either  $E_y$  or  $B_x$  or both?' This issue will be conferred below later, in the paragraph with layered earth discussion.

The fields  $B_x$  and  $E_y$  both are sensitive (Fig 5) to the conductivity variation of the oceans. The observations with several oceans differing in their conductivity suggests that at frequencies with  $\delta < d$ , in general, the variation of ocean conductivity is negligible but at frequencies with  $\delta > d$ , the more conductive ocean increases the strength of  $E_y$  and  $B_x$  field.

The average depth of Atlantic, Pacific and Indian Ocean is respectively 3926 m, 4282 m and 3963 m. Moreover, from place to place the oceanic depths are quite diverse and therefore the significance of deepening of the ocean is studied (Fig 4). For a fixed observation depth in ocean, the gradual increase in the strength of  $E_y$  and decrease in the strength of  $B_x$  is the consequence of deepening of the ocean. The component  $E_y$  and  $B_x$ show strongest and weakest strength respectively for the deepest ocean and vice-versa. For frequencies for which the skin depth is smaller than the ocean depth (i.e.  $\delta < d$ ), the further deepening of the ocean (greater than skin depth) is ineffective and therefore the field value may saturate with further deepening.

Another important parameter is the ocean wave velocity. The study suggests (Fig 5) that any bias in velocity value may cause same bias in field strength. The change in velocity does not modify the pattern with depth, but causes a static shift in the field.

Expression for layered Green's function has allowed us to study the EM field variation for different layers with in the earth. The layered model involves both resistive and conductive layers. Clearly, electric field  $E_y$  does not see the layered boundaries (Fig 6) although the magnetic field clearly sees it by offering a change in the slope of  $B_x$  at the boundaries.

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