Invariants of rotation of axes and indicators of dimensionality in magnetotellurics

F.E.M. Lilley[1] and J.T. Weaver[2]

[1] *Research School of Earth Sciences, Australian National University, Canberra, ACT 0200, Australia.* email: ted.lilley@anu.edu.au

[2] *School of Earth and Ocean Sciences, University of Victoria, Victoria, BC 5001, Canada.* email: jtweaver@uvic.ca

SUMMARY

A magnetotelluric tensor from a particular site is taken as an example and analysed in terms of invariants of rotation of the measuring axes. The invariants presented range from the results of principal value decompositions of the real and quadrature parts taken separately to the results of phase tensor analysis. Attention is paid especially to those invariants which indicate dimensionality. Estimates are also obtained for 2D strike angle.

Key words: magnetotellurics, dimensionality, invariants, Mohr diagrams

1 INTRODUCTION

Central to a magnetotelluric study of Earth structure is the determination, from field observations at an array of sites, of values of the magnetotelluric impedance tensors for those sites. Often the interpretation of such observed tensors is straight-forward, enabling the magnetotelluric study to proceed to completion.

Sometimes, however, individual sites may appear anomalous, and need extra attention before their interpretation can proceed. In such cases, calculating and displaying invariants of rotation may be helpful in understanding perplexing characteristics. The present paper gives graphical analyses of a selected example. While the techniques may be most useful in complicated cases, the example presented is relatively simple, as a simple example makes a good introductory case.

The example is presented against a wider background. The procedure for 1D inversion is always based on an invariant (the observed 1D impedance). Similarly 2D inversion is commonly based on the TE (E-pol) and TM (B-pol) impedances, which in this paper are emphasized as invariants of rotation. As the subject of magnetotelluric interpretation advances further into 3D inversion and modelling, the question of which parameters to invert, from a wide range of possible candidates including notably invariants, may be expected to need frequent re-visiting.

2 INVARIANTS OF ROTATION

The significance of invariants of rotation in magnetotelluric interpretation has been recognized for some time (Ingham 1988; Park & Livelybrooks 1989; Fischer & Masero 1994; Lilley 1998). In recent years Szarka & Menvielle (1997) and Weaver et al. (2000) have investigated sets of seven invariants which, together with an eighth value in the form of a geographic bearing, have been needed to fully describe a complex magnetotelluric tensor of eight elements.

The development of phase tensor analysis by Caldwell et al. (2004), and see also Bibby et al. (2005), led Weaver et al. (2003) to present and discuss three invariants (J1, J2 and J3) which arise in phase tensor analysis. Weaver et al. (2003) was reprinted as Weaver et al. (2006).

The recognition that just three invariants carry much important information in many practical situations is now explored in the example of this paper. The three invariants are calculated and displayed as functions of period. For comparison, a variety of other invariants are also displayed, notably the principal decomposition values of Lilley (1998), and the seven invariants of Weaver et al. (2000).

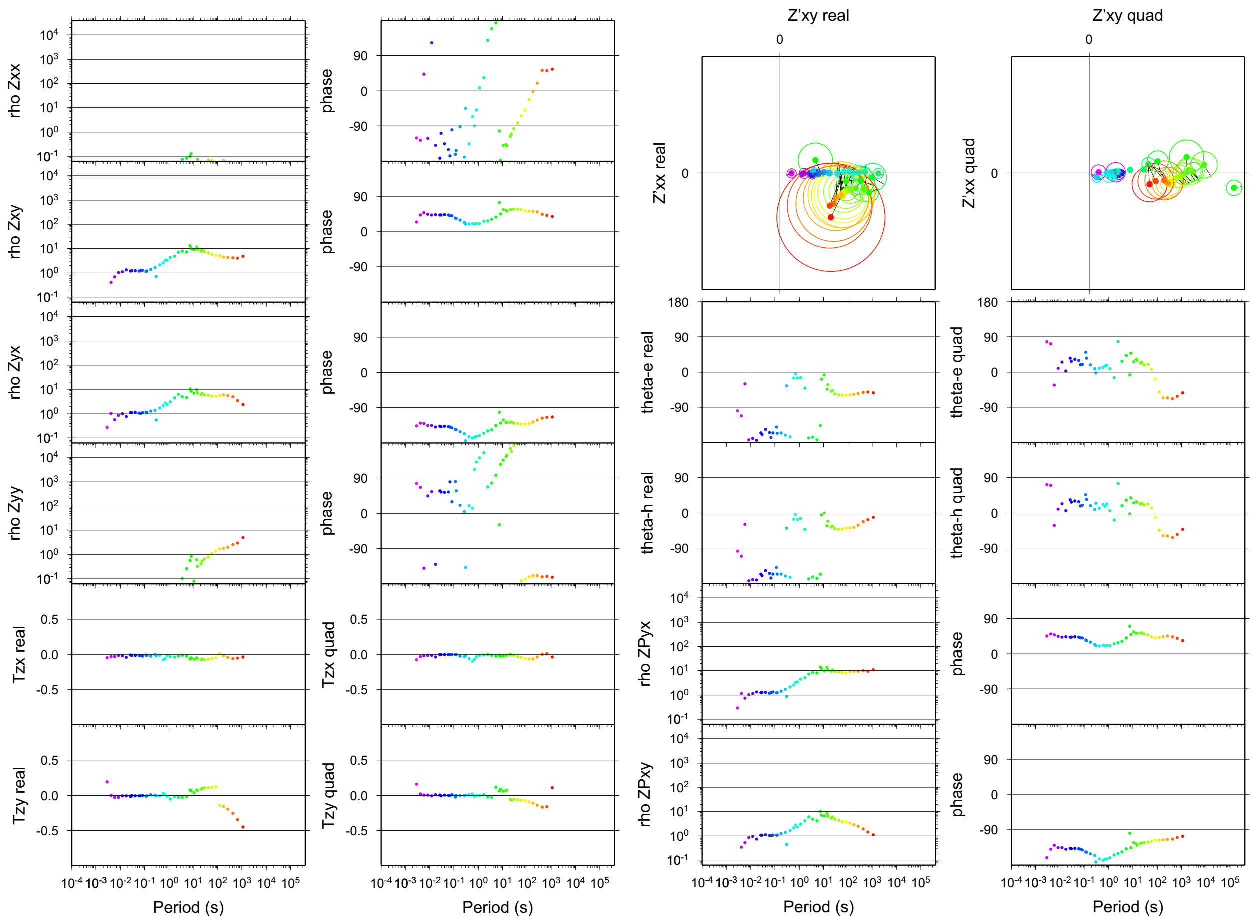

Figure 1. The mimic003 data and their basic decomposition. The units of apparent resistivity (rhoZxx, rhoZxy, rhoZyx, rhoZyy, rhoZPyx and rhoZPxy) are ohm m, and angles of phase and direction are given in degrees. For the Mohr circles, each tensor element value has been scaled by multiplication by the square root of the period, to make the plot of a set of circles more compact.

3 THE EXAMPLE FROM AUSTRALIA

3.1 Data and basic decomposition

The example is from a sedimentary basin in Australia. It is site mimic003 in the Magnetotelluric Investigation of the Mt Isa Crust (MIMIC) experiment of 1997 (Wang 1998; Lilley et al. 2003).

There are two figures presenting results for this site. The first (Figure 1) shows, in its far-left-hand panel, apparent resistivity and phase values calculated from the Zxx, Zxy, Zyx and Zyy tensor elements as observed. The adjoining graphs in the centre-left panel show corresponding phase values (the Tzx and Tzy graphs are not part of the present discussion). The rhoZxy and rhoZyx amplitudes are appropriate for a sedimentary basin, being similar and showing an increase with period (and depth) from a conductive surface layer; they diverge at the longest periods. The Zxy and Zyx phases are in the appropriate quadrants, and again differ from each other only at long periods, consistent with the 1D sedimentary basin where they were recorded. The rhoZxx and rhoZyy amplitudes are generally small, and the phase values generally scattered, again consistent with 1-D behaviour (except at the long-period end of the spectrum, where departure from one-dimensionality below the sedimentary basin is detected).

The centre-right and far-right panels for this figure show, at the top, Mohr circles for the magnetotelluric data. These circles are generally of small diameter (scaled by the distance of the circle centre from the origin of the plot), and are centred on or near the horizontal axes, again indicating one-dimensionality of electrical conductivity structure. In the circles, invariants of rotation of the measuring axes become evident. For example the centres of the circles are fixed by the observed data, and would not change even were the measuring axes to be rotated and so aligned differently.

In the case of an ideal 2D structure, the E-pol and B-pol impedances are given by the two points where the circle (itself now centred on the horizontal axis) cuts the horizontal axis. Once determined, these points of intersection which indicate E-pol and B-pol values do not change with measuring axis rotation, and are invariants of rotation.

The panels below the circles show the results (centre-right : real; far-right : quad) of principal value decompositions of the magnetotelluric tensor, taking real and quad parts separately (Lilley 1998). The situation is evisaged where an ideal two-dimensional tensor is measured using axes (say aligned north and east) relative to which the geologic strike has bearing theta-h, and the electric field is distorted from its 2D direction by an angle (theta-e minus theta-h). Then rotating the magnetic axes by angle theta-h and the electric axes by angle theta-e recovers the original two-dimensional tensor, and its E-pol and B-pol values.

Using equations (107) - (114) of Lilley (1998), the panels show the theta-e and theta-h values for the mimic003 example, and below them, values of rhoZPyx and rhoZPxy respectively (centre-right : amplitude; far-right : phase). In the calculation of the rhoZPyx values, the real and quad parts of ZPyx have first been combined in the usual way to give ZPyx amplitudes, even though the real and quad parts of ZPyx may have resulted from different values of theta-h and theta-e. The same applies to ZPxy and rhoZPxy. For the simple 2D distortion case described, such rhoZPyx and rhoZPxy results are the undistorted E-Pol and B-pol values (possibly interchanged).

Examining the results plotted for the mimic003 example, it is evident that neither theta-e nor theta-h are well determined for most of the period range (stable values start to become evident for $T > 10$ s). This behaviour is consistent with 1D structure.

The plots for rhoZPyx and rhoZPxy are well behaved in both amplitude and phase. They agree for most of the spectrum, again indicating one-dimensionality; however at periods above 10 s they begin to diverge, showing the conductivity structure becoming more complicated, as already noted.

3.2 Invariants as a function of period

The second figure (Figure 2) moves to the calculation and presentation of the seven invariants of rotation of Weaver et al. (2000), denoted (in the far-right and centre-right panels) as I1 to I7. Two supplementary invariants are also included, I and I0 (Weaver et al. 2003, 2006). The seven invariants (I1 to I7) monitor the dimensionality of the impedance tensor as a function of period. Specifically, I1 and I2 gauge the scale of the tensor: see equation (24) of Weaver et al. (2000). The quantities I3 and I4 are dimensionless, vanish for 1D, and otherwise gauge the extent of two-dimensional anisotropy: see equations (26) and (27) of Weaver et al. (2000). The quantities I5, I6 and I7 (also dimensionless) gauge three-dimensionality, see equations (51) and (52) of Weaver et al. (2000). Of the supplementary invariants in Figure 2, I is related to I1 and I2. The supplementary invariant I0 is related to I7, in that the vanishing of I0 implies that I7 is undefined.

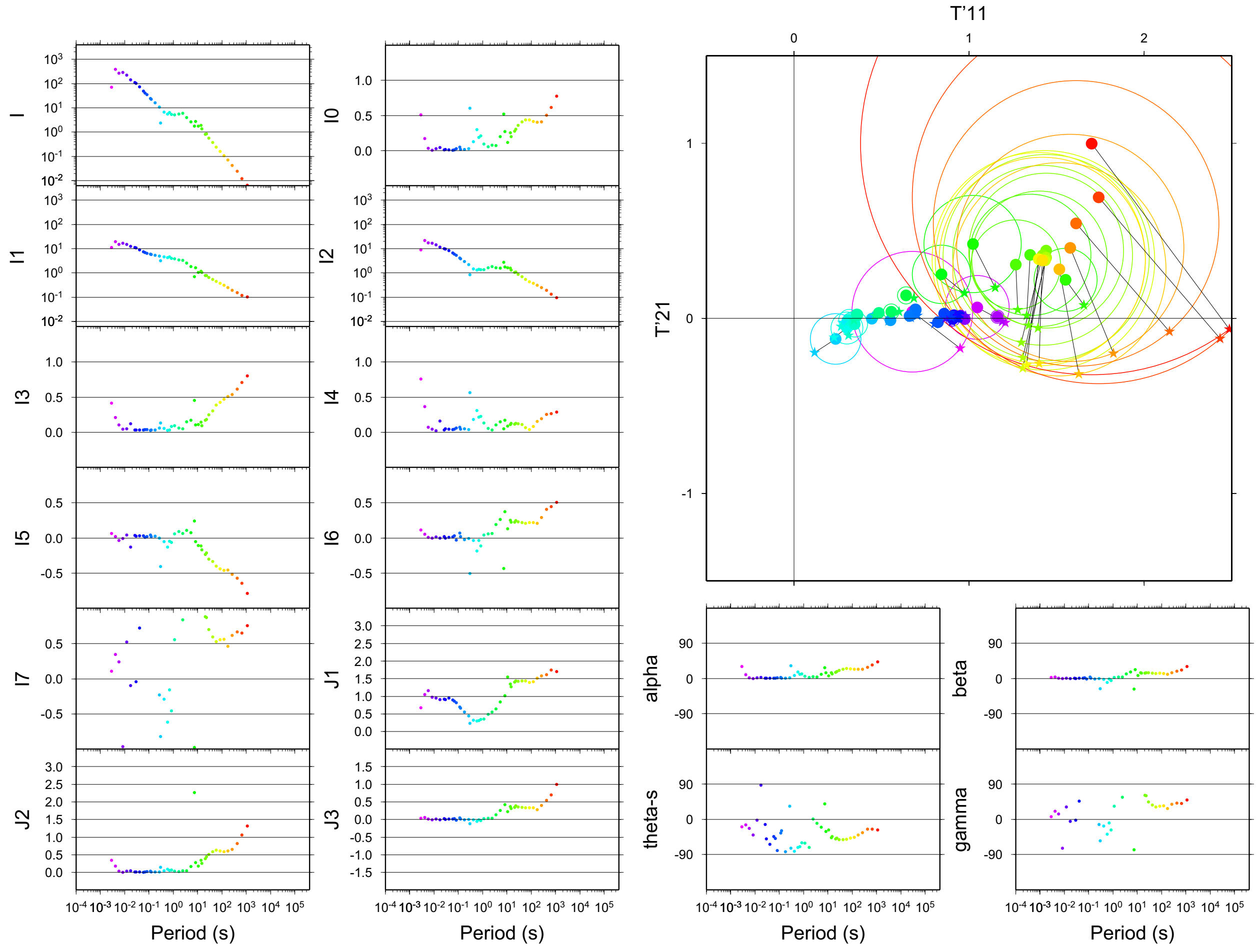

Figure 2. Invariants as a function of period for the mimic003 data. The units of I1 and I2 are those of the observed tensor elements, and I has those units squared. The invariants I0, I1 to I7, and J1 to J3 are dimensionless, as are the quantities T'11 and T'21, plotted to give the Mohr circles. For 1D data (points on the horizontal axis) T'11 is the trigonometric tangent of the phase value of the magnetotelluric impedance.

Thus in Figure 2, invariants I1 and I2 show a common, well-behaved smooth decrease with increasing period (which is seen also in I). Invariants I3 and I4 show values near zero for most of their period range, a clear indication of one-dimensionality in the observed data. Further I5 and I6 also show values near-zero for most of their period range, indicating absence of three-dimensionality, and I7 is correspondingly indeterminate and unstable.

Three related independent invariants, J1, J2 and J3, as introduced by Weaver et al. (2003, 2006) and which can be expressed in terms of I, IO, I1, I3 and I7, are next considered. They are closely related to the phase tensor analysis of Caldwell et al. (2004) and summarise neatly the extent to which the dimensionality of the data is 1D, 2D or 3D. Their values are plotted at the bottom of the far-left and centre-left panels. The values of J2 and J3 of zero for periods less than 10 s demonstrate very clearly where the data are one-dimensional.

These phase tensor invariants may also be displayed in Mohr circles, as demonstrated by Weaver et al. (2003, 2006). Such Mohr circles are presented at the top of the right-hand side of Figure 2. The horizontal distance from the origin to the centre of a circle is J1, and is a basic scale for the phase tensor. The radius of a circle is J2, and is a measure of the two-dimensionality of the data. The offset of the circle centre from the horizontal axis is J3, and is a measure of the three-dimensionality of the data.

Inspection of Figure 2 shows that indeed at short periods the circles are points close to the horizontal axes, and so are 1D in character. Two-dimensionality and three-dimensionality enter the data together as period increases to greater than 10 s. These characteristics, evident in the circle plots, are consistent with the numerical values of J1, J2 and J3 plotted at the bottom of the left-hand panels.

The angles alpha, beta and gamma, presented below the Mohr circles in Figure 2, are auxiliary to the analysis (Weaver et al. 2003, 2006). Alpha is closely related to J2, and is zero when J2 is zero. Beta is the arctangent of (J3/J1), and is zero when J3 is zero. Gamma is the arcsine of (J3/J2), and is unstable for a one-dimensional situation, when both J3 and J2 are small.

The angle theta-s, plotted below alpha, is presented by Weaver et al. (2003, 2006) as the angle of 2D strike, when such a strike exists. It is equivalent to the Bahr angle (Bahr 1988). In the present example only for periods greater than 10 s is this quantity at all well determined, and then, as has been seen, three-dimensional characteristics enter the data at the same time as two-dimensional characteristics. However, taken as the strike angle for a 2D model, the values plotted for theta-s may be compared with the values for theta-h (real and quad) in Figure 1. It can be seen there is consistency between these different determinations of 2D strike direction.

4 CONCLUSIONS

The graphical analysis of rotationally invariant quantities of a magnetotelluric tensor may help the interpreter to understand particularly some 3D behaviour. Such understanding may be crucial in deciding when simpler 2D or even 1D modelling and interpretation may be appropriate. Various estimates of 2D strike angle also arise in invariant analysis, and may be useful in the modelling and inversion of observed data.

Of the many invariants which may be calculated and examined, three which arise from phase tensor analysis have great appeal. In a direct way they scale and demonstrate 1D, 2D and 3D behaviour, respectively.

5 ACKNOWLEDGEMENTS

Many colleagues have helped the authors develop the ideas in the present paper, and are thanked for their comments, criticisms, and discussions. Professor Joseph Pek and colleagues, and the EMTF group, are thanked for their hospitality at the most valuable and enjoyable EMTF07 meeting.

REFERENCES

Bahr, K., 1988, Interpretation of the magnetotelluric impedance tensor: regional induction and local telluric distortion, *J. Geophys.*, **62**, 119 – 127.

Bibby, H. M., Caldwell, T. G., & Brown, C., 2005, Determinable and non-determinable parameters of galvanic distortion in magnetotellurics, *Geophys. J. Int.*, **163**, 915 – 930.

Caldwell, T. G., Bibby, H. M., & Brown, C., 2004, The magnetotelluric phase tensor, *Geophys. J. Int.*, **158**, 457 – 469.

Fischer, G. & Masero, M., 1994, Rotational properties of the magnetotelluric impedance tensor: the example of the Araguainha Crater, Brazil, *Geophys. J. Int.*, **119**, 548 – 560.

Ingham, M. R., 1988, The use of invariant impedances in magnetotelluric interpretation, *Geophys. J. R. Astron. Soc.*, **92**, 165 – 169.

Lilley, F. E. M., 1998, Magnetotelluric tensor decomposition: Part I, Theory for a basic procedure, *Geophysics*, **63**, 1885 – 1897.

Lilley, F. E. M., Wang, L. J., Chamalaun, F. H., & Ferguson, I. J., 2003, Carpentaria Electrical Conductivity Anomaly, Queensland, as a major structure in the Australian Plate, in *Geological*

Society of Australia Special Publication No. 22, Evolution and Dynamics of the Australian Plate, edited by R. R. Hillis & R. D. Muller, pp. 141 – 156.

Park, S. W. & Livelybrooks, D. W., 1989, Quantitative interpretation of rotationally invariant parameters in magnetotellurics, *Geophysics*, **54**, 1483 – 1490.

Szarka, L. & Menvielle, M., 1997, Analysis of rotational invariants of the magnetotelluric impedance tensor, *Geophys. J. Int.*, **129**, 133 – 142.

Wang, L. J., 1998, *Electrical Conductivity Structure of the Australian Continent*, Ph.D. thesis, The Australian National University.

Weaver, J. T., Agarwal, A. K., & Lilley, F. E. M., 2000, Characterization of the magnetotelluric tensor in terms of its invariants, *Geophys. J. Int.*, **141**, 321 – 336.

Weaver, J. T., Agarwal, A. K., & Lilley, F. E. M., 2003, The relationship between the magnetotelluric tensor invariants and the phase tensor of Caldwell, Bibby and Brown, in *Three-Dimensional Electromagnetics III*, edited by J. Macnae & G. Liu, no. 43 in Paper, pp. 1 – 8, ASEG.

Weaver, J. T., Agarwal, A. K., & Lilley, F. E. M., 2006, The relationship between the magnetotelluric tensor invariants and the phase tensor of Caldwell, Bibby and Brown, *Explor. Geophys.*, **37**, 261 – 267.