MT Robust Remote Reference Processing revisited

Thomas Krings\textsuperscript{1,2}, Oliver Ritter\textsuperscript{2}, Gerard Muñoz\textsuperscript{2}, Ute Weckmann\textsuperscript{2}

\textsuperscript{1} Westfälische Wilhelms-Universität Münster
\textsuperscript{2} GeoForschungsZentrum Potsdam

contact: t_krings@gfz-potsdam.de

1 Introduction

Magnetotelluric (MT) data is often heavily affected by cultural noise, particularly when measurements are carried out close to inhabited areas. The geothermal test site Groß Schönbeck is located only 25 km north of the outskirts of Berlin and consequently the MT data acquired in 2006/2007 is contaminated by noise. Figure 1 reveals obvious artificial features in the apparent resistivities and phase curves at periods of 10 s and 100 s.

To overcome or at least reduce the influence of noise several techniques can be applied:

- Robust Statistics to minimize the influence of outliers
- Remote reference processing to minimize the impact of local noise

The robust processing code based on Junge (1993) and Ritter \textit{et al.} (1998) has been used successfully to process single site data but it is not straight forward to adopt the scheme for remote reference processing.

2 Remote Reference

For MT data processing bivariate equations of the following type have to be solved:

\[ Z_i = a \cdot X_i + b \cdot Y_i + r_i \] (1)
Figure 1: Typical single site processing results for a station from the 2007 I-GET magnetotelluric survey (see also Muñoz et al., this volume), showing apparent resistivities, phases and induction arrows.
where $a$ and $b$ denote the frequency dependent, complex valued transfer functions which do not depend on time. $X_i, Y_i$ and $Z_i$ refer to observations of electromagnetic field components recorded at different time segments $i$. $X$ usually refers to the magnetic field component in x-direction $B_x$, $Y$ to the magnetic field in y-direction $B_y$ and $Z$ to either one of the horizontal electric field components $E_x$ or $E_y$, or the vertical magnetic field $B_z$. The noise term $r_i$ is attributed to $Z_i$. To solve for the transfer function $a$ a least squares approach is used:

$$a = \frac{\langle YY^* \rangle \langle ZX^* \rangle - \langle YX^* \rangle \langle ZY^* \rangle}{\langle XX^* \rangle \langle YY^* \rangle - \langle XY^* \rangle \langle YX^* \rangle}$$

(2)

where $\langle ZX^* \rangle = \sum_i (Z_i X_i^*)$. In the remote reference formulation auto spectra of the local fields are replaced by cross spectra between local and remote (index $R$) fields:

$$a_R = \frac{\langle YY_R^* \rangle \langle ZX_R^* \rangle - \langle YX_R^* \rangle \langle ZY_R^* \rangle}{\langle XX_R^* \rangle \langle YY_R^* \rangle - \langle XY_R^* \rangle \langle YX_R^* \rangle}$$

(3)

Any uncorrelated noise between local and remote station cancels out.

### 3 Robust Statistics

Robust statistics is based on the stability theory of statistical methods and examines the influence of deviations (outliers) from a parametric model. Basic concepts include the breakdown point of a robust method as an indicator for its stability (robustness) against outliers. The ordinary least squares (OLS) approach is an optimum solution for Normal distributed data but tends to fail for long tailed distributions, i.e. distributions with an increased probability for outliers. Because ordinary least squares methods are well established and easy to compute, we use a weighted least squares approach (WLS) with weights $w_i$ based on Huber’s M-estimates (Huber, 1964). The actual weights are calculated to minimize the residuals $r_i$, by down-weighting outliers:

$$OLS : \sum_i r_i^2 \rightarrow min$$

(4)

$$WLS : \sum_i w_i r_i^2 \rightarrow min$$

(5)

The squared residual is given by the following expression:

$$r_i^2 = |Z_i - a \cdot X_i - b \cdot Y_i|^2$$

(6)
4 Modifications and Results

In the remote reference case it is not straightforward to define the residual \( r \). Obviously the remote reference transfer functions \( a_R \) and \( b_R \) have to be taken into consideration. Since these transfer functions are calculated using cross spectra between local and remote fields (see eq. 3) they do not resemble the residual defined in equation 6, which only depends on local spectra. To obtain a suitable expression the residual can be redefined \textit{ad hoc} to include also the horizontal magnetic spectra from the remote site:

\[
|r_{RR,i}|^2 \approx \frac{\langle rX^{R,*} \rangle - \langle rX^{R,R} \rangle}{\langle X^{R,Y^{R,*}} \rangle} = \frac{\langle ZX^{R,*} - a_R \cdot XX^{R,*} - b_R \cdot YY^{R,*} \rangle - \langle ZY^{R,*} - a_R \cdot XX^{R,*} - b_R \cdot YY^{R,*} \rangle}{\langle X^{R,Y^{R,*}} \rangle} \tag{7}
\]

Figure 2 shows processing results before these modifications were applied to the code. Obviously the results are severely distorted. The usage of the new definition of the residual is a major improvement (see Figure 3). However, the results are still not quite as good as desirable, because apparent resistivities and phases are not well resolved in the so-called dead band around 10 s.

Obviously the robust scheme is not able to identify all of the “bad” data. To obtain a better understanding of the actual weighting process Figure 4 shows the transfer functions \( a_R \) (upper left quadrant) and \( b_R \) (upper right quadrant) for the case \( Z = E_x \) at a period of \( T = 8 \) s. For this period range the processing cannot reconstruct reliable values for apparent resistivities and phases. The picture also shows the residuals, i.e. the misfit for each data event (lower left quadrant) and the corresponding weights (lower right quadrant). The robust scheme consists of two algorithms, which are applied sequentially. Where the first robust algorithm calculates the residuals by evaluating the transfer functions for each time segment separately, the second algorithm computes global transfer functions which are averaged over all time segments. The black dots indicate an event with a weighting factor above 0.7. Red dots mark events that were down-weighted by the first robust algorithm by a factor of 0.7 or less. Events that were not already down-weighted by the first algorithm but eventually by the second algorithm (by a factor less than 0.7) are coloured in green.

When examining the distribution of the transfer functions, it is obvious that some data points located far away from the centre of the distribution are not down-weighted. The \( a_R \) is more affected than the \( b_R \) component. Other events, although being strongly biased by noise, are not at all recognized
Figure 2: Using the old definition for the residuals leads to strongly distorted results for the remote reference processing.

Figure 3: Robust remote reference processing result using the new definition of the residuals in equation 7.
by the weighting scheme. The reason could be that the calculation of the residual is based on the simultaneous estimation of both transfer functions. This means that if one transfer function is well determined and the other one poorly, the residual can still be below the threshold for down-weighting. Thereby, huge outliers in either one of the two connected transfer functions can be masked if one of the transfer functions is well determined.

The picture in the lower left quadrant shows that most of the large residuals are already recognized by the first algorithm, whereas the second mainly works as a refinement to this “preselection”. The graphical display of the weights confirms, that the first algorithm is responsible for most of the down-weighting. There is a transition zone between supposedly “good” data (weighting factor above 0.7) and “bad” data (weighting factor below 0.7). However, hardly any weights are between 0.7 and 0.95. The causes for this abrupt transition are unclear.

Figure 4: Transfer functions $a_R$, $b_R$, residuals and weights for the period of $T = 8$ s from Groß Schönbeck experiment (station 10).

5 Conclusions

The definition of a new residual (along with other modifications to the code) significantly improved the robust remote reference scheme which is now ca-
pable of efficiently suppressing noise in the data efficiently. However, there are still problems to achieve good data quality of very disturbed datasets particularly in the period range of the dead band (around 1s).

A closer inspection of the weighting process revealed that many outliers remain undetected. This could be due to the fact that we solve a bivariate equation, where one well estimated transfer function can mask an associated severely distorted transfer function.

References

