Contact impedance of grounded and capacitive electrodes

Andreas Hördt
Institut für Geophysik und extraterrestrische Physik, TU Braunschweig

Abstract

The contact impedance of electrodes determines how much current can be injected into the ground for a given voltage. If the ground is very resistive, capacitive coupling may be superior to galvanic coupling. The standard equations for the impedance of capacitive electrodes assume that the halfspace is an ideal conductor. Over resistive ground at high frequencies, however, the contact impedance will depend on the electrical properties, i.e. electrical conductivity and permittivity, of the subsurface. Here, I review existing equations for the resistance of a galvanically coupled, spherical electrode in a fullspace. I extend the theory to the general case of a sphere in a spherically layered fullspace which may display both galvanic and capacitive coupling.

For a capacitively coupled electrode, the common assumption of an ideally conducting fullspace (or halfspace) breaks down if the displacement currents in the fullspace become as large as the conduction currents. For a moderately resistive medium with 1000 Ωm this is the case for frequencies larger than 100 kHz. For very high resistivities around 1 MΩ, the transition frequency reduces to 100 Hz. Thus, in principle, one may determine electrical resistivity and permittivity by measuring magnitude and phase of the electrode contact impedance.

Introduction

DC resistivity measurements are usually carried out with four electrodes. This way, the ratio between measured voltage and injected current is independent of the grounding resistance of the electrodes. However, calculation or estimation of the electrode resistance may be important in some situations. If the ground is very resistive, technical issues may limit the current that can be injected into the ground. When trying to decrease contact resistance, for example by watering electrodes, the exact dependence on ground resistivity or geometry is important to find an optimum strategy. Finally, the contact resistance itself might be used to obtain information on the ground resistivity (Dashevsky et al., 2005).

For galvanicall coupled electrodes, equations descibing the injected current as function of voltage have been derived for different electrode geometries by Krajew (1957). Capacitive electrodes normally consist of sheets close to the ground with no direct contact. They are used with an alternating current of sufficiently high frequency such that the impedance is sufficiently low. They may be particularly useful if the ground is very resistive and galvanic coupling is not feasible, or if fast measurents with a moving system are to be carried out. Kuras et al. (2006) describe the theory behind 4-point resistivity measurements with capacitive electrodes and discuss the conditions under which inductive currents may be ignored. To estimate the contact resistance of capacitive electrodes, the halfspace is normally assumed to be an ideal conductor. Over very resistive ground, however, the assumption of an ideal conductor is no longer valid, and the contact resistance of capacitive electrodes will depend on electrical conductivity and dielectric permittivity of the halfspace.

Here, I review the equations for galvanically coupled electrodes and extend the theory to capacitively coupled spheres in a fullspace. I investigate under which conditions the assumption of an ideally conducting halfspace breaks down and 2-point measurements might be feasible to determine conductivity and dielectric permittivity of the ground.

The basic setup is sketched in figure 1. A DC voltage is applied to galvanically coupled electrodes (top panel) or an AC voltage to capacitively coupled electrodes (bottom panel). The aim is to derive equations for the resistance *R*, required to calculate the current *I* from the applied voltage *U* via:

$$R = U / I \qquad (1)$$

where *R* depends on resistivity for galvanic coupling, and on resistivity and electric permittivity for capacitive coupling.

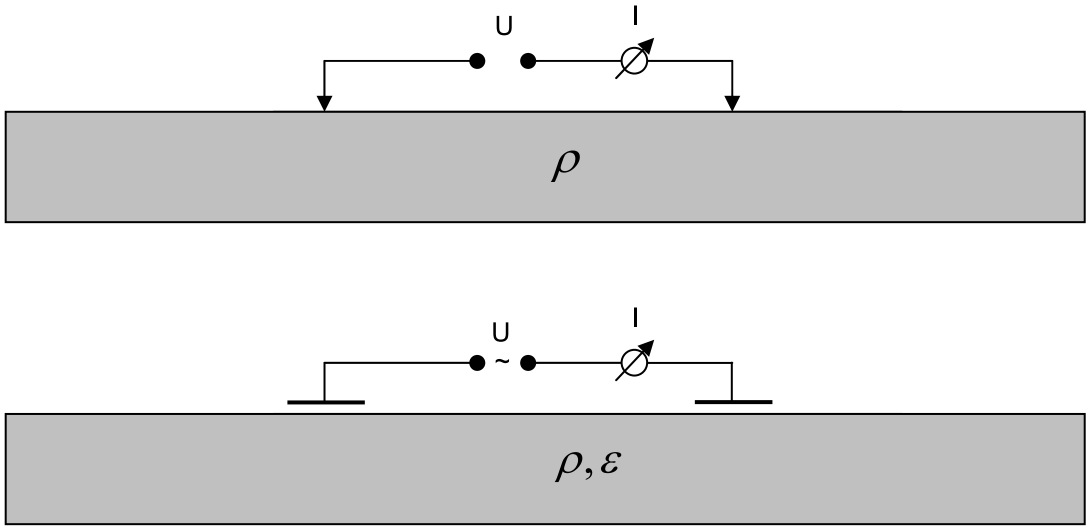

Figure 1: Sketch of the basic setup. Top panel: DC voltage applied to galvanically coupled electrodes. Bottom: AC voltage applied to capacitively coupled electrodes.

Galvanically coupled spherical electrode in fullspace

The calculation of the resistance of arbitrary electrodes over a halfspace depends on the shape of the electrodes and requires numerical solution. Therefore, I simplify the problem by considering spherical electrodes in a fullspace. This strongly deviates from the situation sketched in figure 1, but in order to obtain physical insight, simple analytic equations are desired. The equation for the contact resistance of a single galvanically coupled spherical electrode in a fullspace was given by Krajew (1957):

$$R = \frac{\rho}{4\pi r_0} \qquad (2)$$

where ρ is the resistivity of the fullspace and r_0 is the radius of the sphere. One important implication is that the resistance is inversely proportional to the radius, and not to the surface of the electrode. This will apply to other types of electrodes as well, in a sense that the spatial dimension of the electrode enters linearly into the resistance. The linear dependence might be counterintuitive, because one could expect the resistance to decrease with the surface area of the sphere. The important point is that the electric field at the surface of the sphere decreases with $1/r_0$, which compensates one spatial dimension, as can be seen from the derivation in appendix 1.

Another useful assumption is that the distance between the two electrodes is large compared to the size of the electrodes. In that case, each electrode may be treated independently. The distance between the electrodes drops out of the equations and the total resistance will simply be the sum of the two single electrode resistances (Krajew, 1957).

Equation (2) can easily be extended to the situation where the electrode is surrounded by spherical shells. The parameters for the case of two spheres, which will be sufficient to describe most of the practical situations, are defined in figure 2:

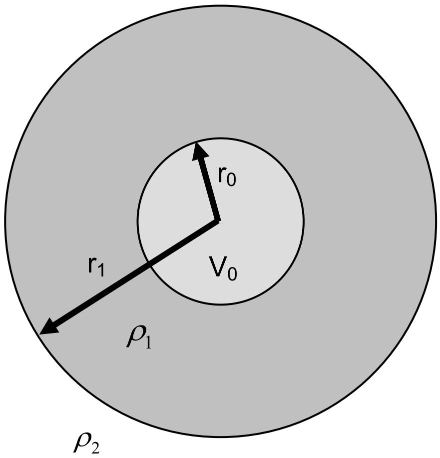

Figure 2: Geometry of a spherical electrode, radius r_0 with potential V_0, surrounded by a sphericall shell with radius r_1 and resistivity ρ_1, in the fullspace with resistivity ρ_2.

The resistance of the spherical electrode is given by:

$$R = \frac{\rho_1}{4\pi r_0}\left(\frac{1-\frac{\rho_1}{\rho_2}+\frac{\rho_1}{\rho_2}\frac{r_1}{r_0}}{\frac{\rho_1}{\rho_2}\frac{r_1}{r_0}}\right) = \frac{\rho_2}{4\pi r_0}\left(\frac{1-\frac{\rho_1}{\rho_2}+\frac{\rho_1}{\rho_2}\frac{r_1}{r_0}}{\frac{r_1}{r_0}}\right) \qquad (3)$$

A derivation slightly deviating from that of Krajew (1957) is given in Appendix 1.

Equation (3) may be used in different forms to study the dependence of resistance on the resistivity distribution of the volume surrounding the electrodes. It is common practice to decrease contact resistance by pouring water into the ground near the electrode, and we may estimate the amounts of water and the resistivity contrast which is required to achieve a certain reduction in resistance. We assume that the water fills a spherical shell of radius r_1 and reduces the resistivity to ρ_1 compared to ρ_2 of the undisturbed formation. The decrease of contact resistance is then expressed as

$$\frac{R}{R_0} = \left(\frac{1-\frac{\rho_1}{\rho_2}+\frac{\rho_1}{\rho_2}\frac{r_1}{r_0}}{\frac{r_1}{r_0}}\right) \qquad (4)$$

where $R_0 = \dfrac{\rho_2}{4\pi r_0}$ (5)

denotes the resistance in a fullspace with resistivity ρ_2, which would exist if no watering was applied.

Figure 3 illustrates the reduction of electrode resistance by a conductive spherical shell surrounding the electrode. The resistance quickly decreases with the size of the conductive shell, but for radii larger than 10 times the electrode size, a further increase is not efficient any more. The behavior with respect to resistivity contast is similar: Once a reasonable resistivity contrast of 1:10 is reached, a further increase does not lead to a significant decrease of resistance.

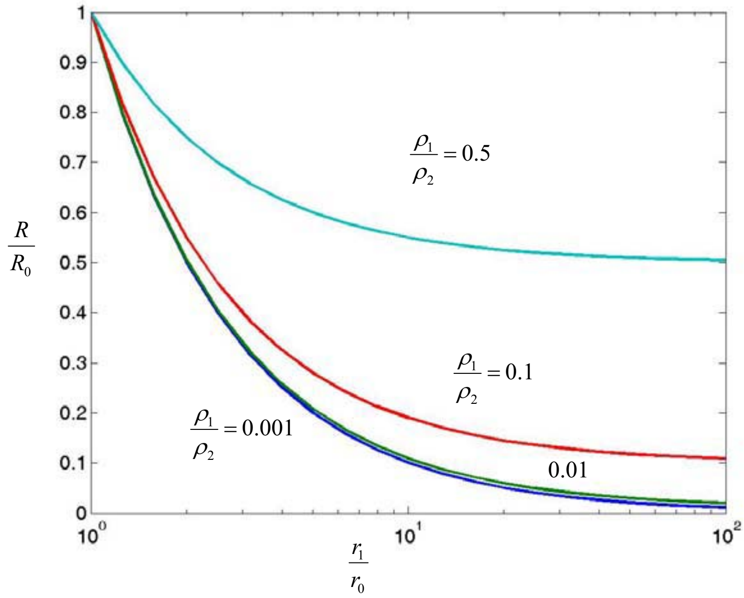

Figure 3: Reduction of electrode resistance as function of radius of the conductive shell for different resistivity ratios between outer fullspace and conductive shell. Note the logarithmic radius axis.

Capacitively coupled sphere

For the capacitively coupled sphere, it is useful to use electrical conductivity instead of resistivity. We may use the same equations derived for the static case if we replace the electrical conductivity σ by a complex conductivity defined by:

$$\sigma^* = \sigma + i\omega\varepsilon \tag{6}$$

where ε is the dielectric permittivity. This substitution is justified in detail in Appendix 2. One assumption which is not expanded on here is that induction effects may be ignored. This aspect was discussed in some detail by Kuras et al. (2006). The complex electrode impedance is obtained by rewriting eq(3) with the substitution defined in eq. (6):

$$Z = \frac{1}{4\pi r_0 \sigma_1^*}\left(\frac{1 - \frac{\sigma_2^*}{\sigma_1^*} + \frac{\sigma_2^*}{\sigma_1^*}\frac{r_1}{r_0}}{\frac{\sigma_2^*}{\sigma_1^*}\frac{r_1}{r_0}}\right) \tag{7}$$

A capacitively coupled electrode may be studied by setting conductivity and relative dielectric permittivity in the inner shell to the values of air (*$\sigma_1=0$, $\varepsilon_{r1}=1$)*. If the fullspace surrounding the electrode is sufficiently conductive, the common ideal conductor assumption will hold, and the resistance will not depend on the electrical parameters of the fullspace. This can be seen by writing eq. (7) in the limit $\sigma \rightarrow \infty$:

$$Z = \frac{r_1 - r_o}{i\omega\varepsilon_o 4\pi r_0 r_1} \tag{8}$$

which may be compared with the impedance of a plate over an ideally conductive halfspace:

$$Z = \frac{d}{i\omega\varepsilon_o A} \tag{9}$$

where *d* is the distance between the halfspace and the plate, and *A* is the area of the plate. Obviously, the thickness of the inner shell (r_1-r_0) corresponds to *d*, and $4\pi\ r_0\, r_1$ corresponds to the area *A*.

However, for a resistive fullspace, this approximation will not be valid any more. The transition is illustrated in figure 4, which shows the resistance for a spherical capacitive electrode with *1mm* separation between electrode and fullspace, calculated from eq. (7). The curve for σ_2=*1 S/m* represents the ideally conducting fullspace. The resistance follows a *1/ω* frequency dependence over the entire frequency range, and does not depend on conductivity or permittivity of the fullspace. The upper limit is set by the curve for very low conductivities *($\sigma_2=10^{-12}$ S/m)* which represents a spherical electrode in the air.

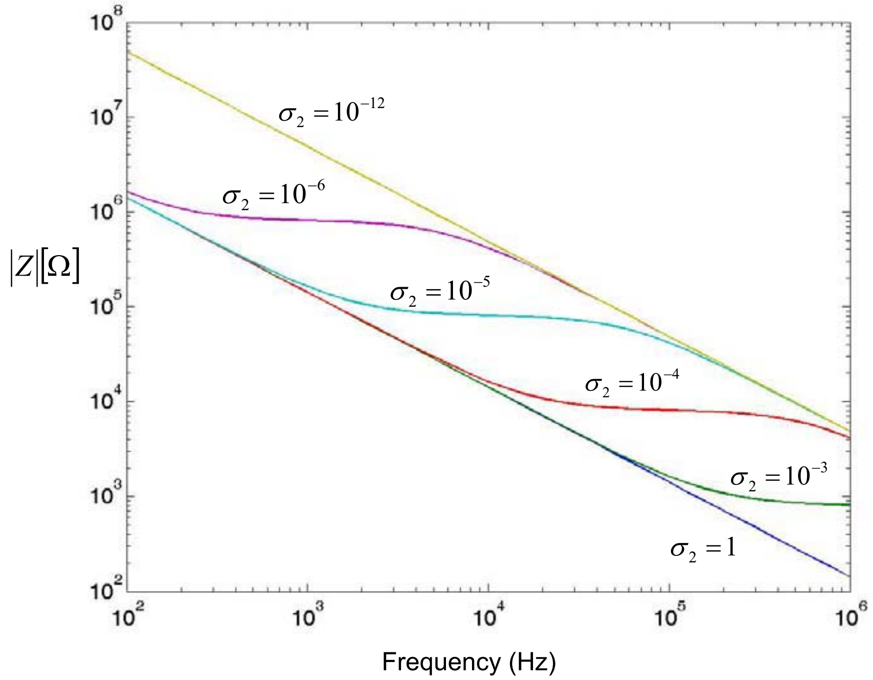

Figure 4: Amplitude of the complex impedance as function of frequency for different electrical conductivities (in S/m) of the fullspace. The radius of the spherical electrode is r_0*=0.1m*, the shell between the fullspace and the electrode is *1 mm* thick *(r_1-r_0=0.001m)*, and the relative permittivity of the fullspace is ε_r*=3*.

If the fullspace is moderately resistive (i.e. $\sigma_2=10^{-3}$ *S/m),* the electrode resistance starts to deviate from the ideal conductor limit at approx. *100 kHz*. If the fullspace is very resistive (i.e (i.e. $\sigma_2=10^{-6}$ *S/m),* the transition starts at relatively low frequencies around *100 Hz*. Of course, the transition frequency corresponds to the point where displacement currents start to become as large as conduction currents. Thus, if a capacitive electrode system is used over permafrost areas, over very dry rock, or on space missions landing on asteroids or comets, the ideal conductor equations will break down.

Figure 5 illustrates the behavior of the phase of the impedance. In the limit of an infinitely conductive or resistive fullspace *(*$\sigma_2=1$ *or* 10^{-12} *S/m)*, the impedance behaves like that of an ideal capacitor, and the phase is -90 degrees. For finite fullspace conductivities, the phase will be sensitive to variations in conductivity (and permittivity, not illustrated), which may in principle be used to determine those paramters. Measuring amplitude and phase of the injected current related to the source voltage gives two equations which are required to solve for the two unknowns σ and ε_r. In practice, however, the additional dependence on the distance between electrode and fullspace or halfspace, and capacitive coupling between cables and the measuring device may create difficulties. Dashevsky et al., (2005) suggested to measure the difference of the impedance for two different heights in order to remove coupling effects, and used this approach to evaluate pavement quality.

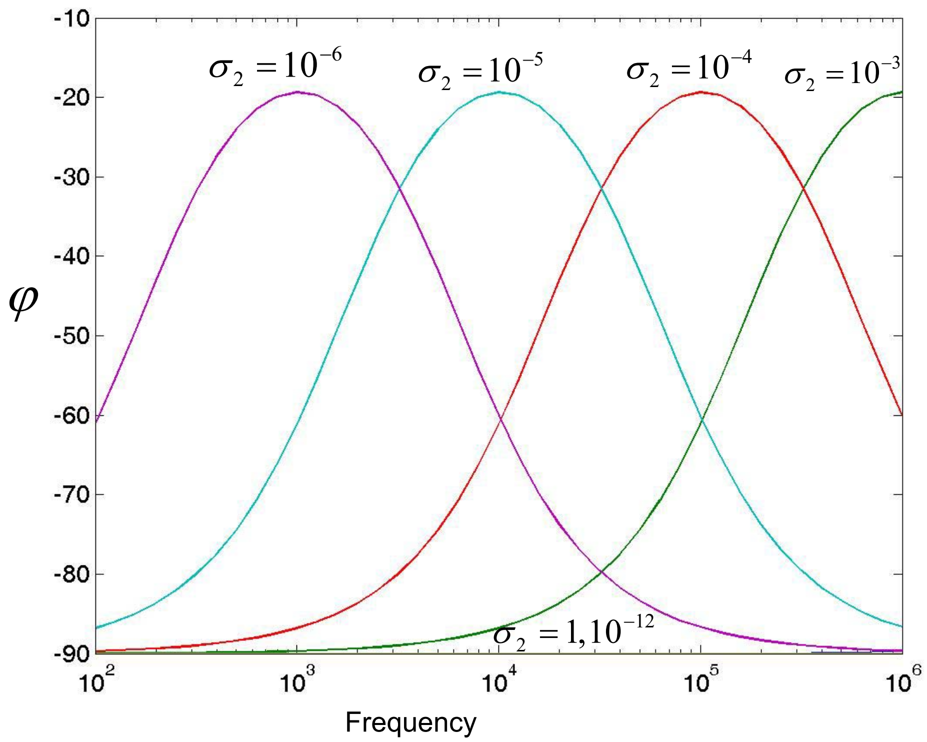

Figure 5: Phase in (degrees) of the contact impedance of a capacitive electrodes for different conductivities in S/m) of the spherical fullspace. Parameters are the same as in figure 3.

Conclusions

For a single, galvanically coupled sphere, the resistance decreases with the radius of the sphere, and not, as one might expect, with the area of the sphere. Thus, if in practice the contact area is increased by using many metal sticks in parallel, the decrease of resistance will be proportional only to the square root of the number of sticks. When reducing contact resistance by watering, there is a saturation effect with respect to both resistivity contrast and volume. Below a resistivity contrasts of 0.1 between water and undisturbed ground, the resistance does not further decrease. Thus, there is no point using excessive amounts of salt to create extremely conductive water.

For capacitively coupled electrodes, the common assumption of an ideal conductor breaks down for resistive ground and high frequencies. Depending on electrode size and geometry, the electrode impedance may be underestimated by two orders of magnitude if the finite conductivity is neglected. In principle, two-point measurements to determine electrical parameters of the subsurface with capacitive electrodes are feasible. However, the penetration depth of such measurements is only in the order of the size of the electrodes. Moreover, distortion effects by capacitive coupling between cables and the measuring device have to be carefully controlled.

References

Dashevsky, Y.A., Dashevsky, O.Y., Filkovsky, M.I., Synakh, V.S., 2005, Capacitance sounding: a new geophysical method for asphalt pavement quality evaluation, J. Appl. Geoph. 57, 95-106.

Kuras, O., Beamish, D., Meldrum, P.I., and Ogilvy, R.D., 2006, Fundamentals of the capacitive resistivity technique. Geophysics, 71, G135-G152.

Krajew, A.P., 1957, Grundlagen der Geoelektrik, VBM Verlag Technik, Berlin.

Appendix 1: Derivation of the electrode resistance for the spherical shell model.

A1.1 Spherical electrode in a homogeneous fullspace

We use the geometry sketched in figure 2. We assume a constant potential V_0 on the spherical electrode with radius r_0. At any distance r from the center of the electrode, the potential for $r>r_0$ must follow:

$$V(r) = V_0 \frac{r_0}{r} \tag{A1}$$

because from potential theory it will decay with $1/r$, and $V(r_0)=V_0$ has to be fulfilled. Therefore, the electric field at r is:

$$E(r) = -\frac{\partial V}{\partial r} = V_0 \frac{r_0}{r^2} \tag{A2}$$

and in particular:

$$E(r_0) = \frac{V_0}{r_0} \tag{A3}$$

This allows us to calculate the current density at the surface of the electrode and the total current by integrating over the area of the sphere:

$$j = \frac{E}{\rho} = \frac{V_0}{\rho r_0} \qquad \text{(A4)}$$

and

$$I = 4\pi\, r_0^2\, j = 4\pi \frac{V_0 r_0}{\rho} \qquad \text{(A5)}$$

Finally, we obtain the electrode resistance from the ratio between potential and current:

$$R = \frac{V_0}{I} = \frac{\rho}{4\pi r_0} \qquad \text{(A6)}$$

which is equal to eq. (2).

A1.2 Spherical electrode within a spherical shell in a fullspace

In order to fulfil Laplace's equation for the potential, in the outer fullspace *($r>r_1$)* it must follow:

$$V(r) = \frac{b}{r} \qquad \text{(A7)}$$

where *b* is a yet unknown constant to be determined from the boundary conditions. Within the inner shell *($r_0<r<r_1$)* we use the form:

$$V(r) = V_0 \frac{r_0}{r} + a\frac{r - r_0}{r} \qquad \text{(A8)}$$

Obviously, this form of *V(r)* fulfils Laplace's equation, and the constant *a* must be determined from the boundary condictions. At the edge outer of the inner shell *($r=r_1$)*, the two potentials must be equal:

$$V(r_1) = V_0 \frac{r_0}{r_1} + a\frac{r_1 - r_0}{r_1} = \frac{b}{r_1} \qquad \text{(A9)}$$

and thus:

$$b = V_0 r_0 + (r_1 - r_0)a \qquad \text{(A10)}$$

The second condition results from the continuity of current density at the boundary. Inside the boundary *($r<r_1$)*, the electric field is:

$$E_1 = -\frac{\partial V}{\partial r} = V_0 \frac{r_0}{r^2} - \frac{r_0 a}{r^2} \qquad \text{(A11)}$$

and outside *($r>r_1$)* it is :

$$E_2 = -\frac{\partial V}{\partial r} = \frac{b}{r^2}$$

Continuity of current density at $r=r_1$ requires that

$$\frac{E_1}{\rho_1} = \frac{E_2}{\rho_2} \tag{A13}$$

and thus

$$\frac{1}{\rho_1}\left(V_0\frac{r_0}{r_1^2} - \frac{r_0 a}{r_1^2}\right) = \frac{1}{\rho_2}\frac{b}{r_1^2} \tag{A14}$$

We now have two equations (A10 and A14) for the two unknowns *a* and *b*, and we obtain

$$a = V_0 \frac{1-\frac{\rho_1}{\rho_2}}{1-\frac{\rho_1}{\rho_2}+\frac{\rho_1 r_1}{\rho_2 r_0}} \tag{A15}$$

The solution allows us to calculate the current density, which may be expressed as:

$$j = j_0 \frac{\frac{\rho_1}{\rho_2}\frac{r_1}{r_0}}{1-\frac{\rho_1}{\rho_2}+\frac{\rho_1}{\rho_2}\frac{r_1}{r_0}} \tag{A16}$$

where

$$j_0 = \frac{V_0}{\rho_1 r_0} \tag{A17}$$

is the current density of the sphere in a fullspace without a spherical shell.
We finally obtain the resistance in the form given in equation (3) through

$$R = \frac{V_0}{I} = \frac{V_0}{4\pi r_0^2 j} \tag{A18}$$

Appendix 2: Derivation of the potential equation in the complex case

Ampere's law states that:

$$rot\underline{H} = \underline{j} + \frac{\partial \underline{D}}{\partial t} \tag{A19}$$

where $\underline{H}$ is the magnetic field, $\underline{j}$ is current density and $\underline{D}$ is the electric displacement. By taking the divergence, we obtain:

$$div\left(\underline{j} + \frac{\partial \underline{D}}{\partial t}\right) = 0 \tag{A20}$$

With

$$\underline{j} = \sigma \underline{E} \qquad \text{(A21)}$$

and transformation to the frequency domain, such that the time derivative becomes a multiplication with *iω*, we get:

$$div(\sigma \underline{E} + i\omega\varepsilon \underline{E}) = 0 \qquad \text{(A22)}$$

If we introduce the complex conductivity

$$\sigma^* = \sigma + i\omega\varepsilon \qquad \text{(A23)}$$

(A22) writes:

$$div(\sigma^* \underline{E}) = 0 \qquad \text{(A24)}$$

Finally, Faraday's law states that

$$rot \underline{E} = -\frac{\partial \underline{B}}{\partial t} \qquad \text{(A25)}$$

If induction effects can be ignored, then

$$rot \underline{E} = 0 \qquad \text{(A26)}$$

and the electric field may be obtained from a scalar potential *V*:

$$\underline{E} = -grad V \qquad \text{(A27)}$$

We thus obtain the basic equation for *V*

$$div(\sigma^* grad V) = 0 \qquad \text{(A28)}$$

which is the basis for the derivation of the electrode resistances. It is identical to the equation used in the static case, the only difference being that it is complex and the DC conductivity was replaced as defined in eq. (A23). Thus, all arguments apply for the complex case as well, and eq. (3) may directly be transferred into eq. (7).