Inversion for Transient ElectroMagnetic (TEM) under inclusion of chebyshev series expansions and lateral constraints

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Abstract

In general TEM data are evaluated by inversion calculations. The found conductivity distributions are commonly ambiguous, especially concerning the noise of the measured data. The use of chebyshev polynomials for the description of the layer boundary and lateral constraints to connect conductivities within one layer can help solving these ambiguities through the assumption of an layered haf space. The investigated methods use an 1D forward calculation, but the inversion algorithm is quasi 2D.

Introduction

Sedimentary soils are often provide an layered subsurface. These subsurfaces are mapped in profile orienteted data and naturally invites a 2D interpretation. Unfortunately 2D forward calculations use huge amounts of computer recources and therefore inversion with this method is time intensive. In many cases where an layered subsurface is suspected, one can force the inversion method into this scheme. A possibility to achieve such results is a Lateral Constrained Inversion (LCI). Often a 1D forward solution with lateral constraints is sufficient to investigate an quasi-layered sedimentary environment (*Auken and Christiansen* (2004)). An alternative, that also requires an layered subsurface, is an inversion with series expansion, where the layer bounderies can be discribed by chebyshev polynomials as basis functions(*Kis* (2002)).The model thicknesses in the model vector substitude with the coefficients of the used polinomials. Chebyshev polynomials have appreciable numerical advantages and are often used in geophysical applications, but the choice of the basis functions mainly depends on the geological model. This work compares the results for these two methods on examples for 1D and 2D synthetic data. The appropriate order of the polinomials and the importance of the profile length will be discussed.

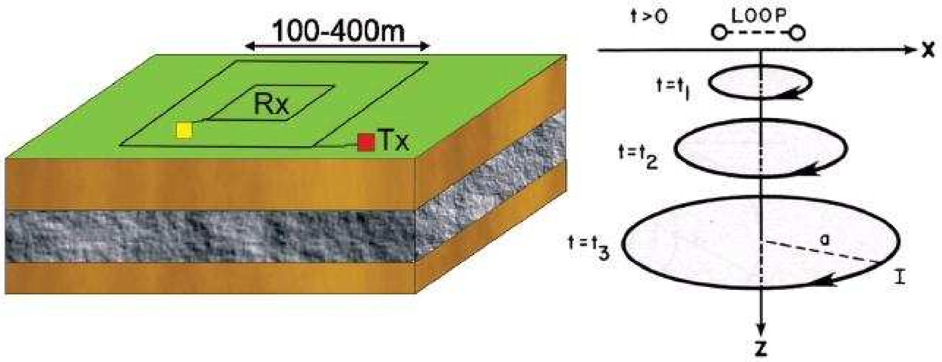

Figure 1: Field setup for Central Loop. *right*: Propagation of ring currents (smoking rings).

TEM

Transient ElectroMagnetic (TEM) is a geophysical method that is capable to derive the conductivity distribution of a subsurface with high resolution. The field setup cosists of a transmitter, driven by an electric current that is suddently switched off, and a reciever that measures the induced voltage of the subsurface currents, caused by the change of the magnetic field (*Helwig* (2003)). Theoretically the switch off process can be described by a Dirac Deltafunction, which contains, applying a Fourier Transformation, all frequencies. Unfortunately, practice relativates this fact. The recieved signal is called transient. Fig. 1 shows the central loop field setup, with a transmitter coil and a reciever coil in the center of the transmitter. Here the current system diffuse down and sidewarts, through smoking rings (*Nabighian and Macnae* (1991)). The setup is called Short Offset TEM (SHOTEM) and has an sheer inductive source. All results in this work referes to this setup. Another setup is the Long Offset TEM (LOTEM), it is e.g. described by *Strack* (1992)

LCI

Geophysical inversion minimizes the misfit between measured and calculated data. The derived cunductivity models are in general ambiguous whitin the data errors. A way to invert TEM data is the Marquardt-Levenberg method.

$$\delta\vec{m} = (\bar{J}\bar{J}^T + \lambda\bar{I})^{-1}\bar{J}^T\delta\vec{d}$$

$\delta\vec{m} = \vec{m} - \vec{m}_0$	$\vec{m}$ model vector
	$\vec{m}_0$ initial model vector
$\delta\vec{d} = \vec{d}_{cal} - \vec{d}_{obs}$	$\vec{d}_{cal}$ calculated data vector
	$\vec{d}_{obs}$ observed data vector

Where the model vectors $\vec{m}$,$\vec{m}_0$ contain the resistivities ρ_i and the layer thicknesses D_i, and the data vectors $\vec{d}_{obs}$,$\vec{d}_{cal}$ contain the voltages. The damping factor λ stabilizes the inversion and determine the step length of one iteration. J is the Jaco-

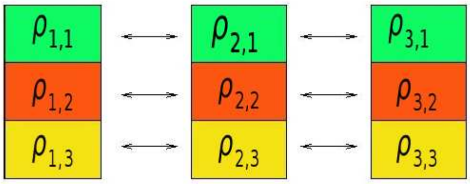

Figure 2: Three sites where the adjacent conuctivities are connected

bian matrix and I the Identity matrix. In this work the LCI, which is developed by *Auken and Christiansen* (2004), applies the Marquardt-Levenberg method to invert a profile, but uses a 1D forward calculation on each site. This is often called 1.5D Inversion. The main idea in LCI is the introduction of the roughening martix R, that is capable to connect model parameters in the inversion scheme:

$$\bar{R}_{n\times m} = \begin{pmatrix} 1 & 0 & \cdots & -1 & \cdots & 0 \\ 0 & \cdots & 1 & \cdots & -1 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 1 & \cdots & -1 \end{pmatrix}$$

In this manner all parameters can be connected, e.g. vertical parameters as well. With the background of an sedimentary quasi-layered subsurface, it is reasonable to apply lateral constraints for adjacent coductivities within one layer, shown in Fig. 2. The invesion problem can than be written:

$$\begin{pmatrix} \bar{J} \\ \bar{R} \\ \bar{I} \end{pmatrix} \delta\vec{m} = \begin{pmatrix} \delta\vec{d}_{obs} \\ \delta\vec{r}_r \\ \delta\vec{m}_{pri} \end{pmatrix} + \begin{pmatrix} \vec{\epsilon}_{obs} \\ \vec{\epsilon}_r \\ \vec{\epsilon}_{pri} \end{pmatrix}$$

where

$$\delta r_p = -\bar{R}\vec{m}_0$$

The first line describes the the inversion as normal with the observation errors ϵ_{obs}. The second line brings the lateral constraints into play, where ϵ_r determines the strenght of the constraint. The third line can be used for a priori information, with $\delta\vec{m}_{pri} = \vec{m}_{pri} - \vec{m}_0$. Where $\vec{m}_{pri}$ contains informations from e.g boreholes and $\vec{\epsilon}_{pri}$ sets the strength for the a priori informations.

Inversion with series expansion

In the inversion with series expansion for chebyshev basis functions the Levenberg-Marquardt method is apllied as well. It also requires a quasi-layered subsurface and inverts profiles in a quasi 2D manner, with 1D forward calculations on each site. But regularisation for the inversion is different. Instead of connecting adjacent conductivities, layer boundaries are described by polinomials (*Kis* (2002)), shown in Fig. 3, therefore the inversion scheme is merely capable to find solutions for an layered subsurface.

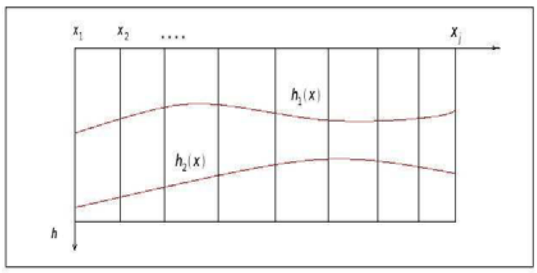

Figure 3: Boundaries are described by polynomials $h_j(x)$. The x_i indicates the measurement sites

The chebyshev polinomials are defined by ($Bronstein\ et\ al.$ (1993)):

$$T_n(x) = cos(n{\cdot}arccos(x)) \quad \text{with} \quad x \in [-1, 1]$$

or can derived by the recursion formula.

$$\begin{aligned} T_n(x) &= 2xT_{n-1} - T_{n-2} \\ \text{for} \quad & T_0(x) = 1 \\ \text{and} \quad & T_1(x) = x \end{aligned}$$

The new model vector $\vec{m}$ contains the coefficients C_i for the chebyshev polinomials in the series expansion on each boundary:

$$\vec{m} = (\rho_{11}, \cdots, \rho_{i1}, \cdots, \rho_{ij}, \cdots, \rho_{mn}, C_{11}, \cdots, C_{1K}, \cdots, C_{(m-1)0}, \cdots, C_{(m-1)K})$$

where:

- $i = 1, \cdots, m$ is the number of the layers and therefore $m - 1$ the number of the boundaries.
- $j = 1, \cdots, n$ is the number of the sites in the profile.
- $k = 0, \cdots, K$ is the order of the chebyshev polinomials and therefore $K + 1$ is the number of coefficients for each polinomial.

1D Syntehetic Data

To verify the LCI and the chebyshev inversions 1D TEM data were generated using the program Emuplus ($Scholl$ (2005)). The distances between the sites are assumed to be equidistant. The data errors are 5% gausian disstributed and increase strongly for late times, as in field measurements. The misfit is calculated by χ, where:

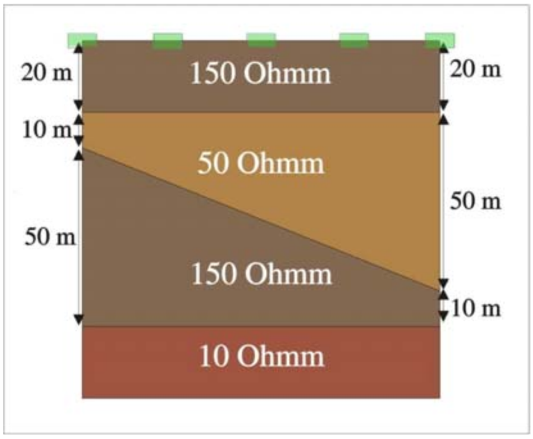

Figure 4: Four layers with five sites (green) and a dipping layer.

$$\chi = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\frac{(d_i^{obs} - d_i^{cal})^2}{\epsilon_i^{obs}}}$$

Fig. 5 shows the result for an inversion without constrains. It shows, that the resistivities ρ_i, especially in layer 3 are hardly represented. The depth on sites 3,4,5 are too low. But within the data errors the model is fitted well. Fig. 6 shows the result after applying lateral constrains. Although the resistivities ρ_i are still too low, the course of the dipping layer is represented much better. The initial model were $\rho_0 = (180, 80, 100, 20)\Omega m$ and $D_0 = (30, 30, 30)m$ in both cases and the inversion was stopped, when $\chi \leq 1$. Like in this example the LCI was always capable to improve the results of 1D data compared to an inversion without constraints.

To apply the chebyshev inversions it is important find out which order of the polinomials is appropriate for inversion prob-

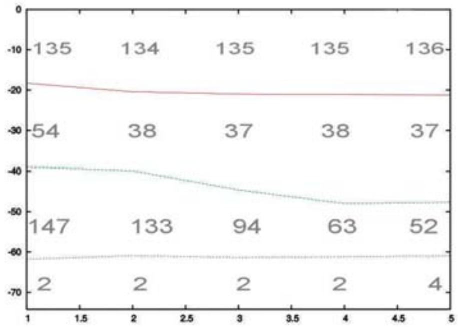

Figure 5: LCI without constraints, resistivity values ρ_i in Ωm. $\chi = 1.00$; 5 iterations.

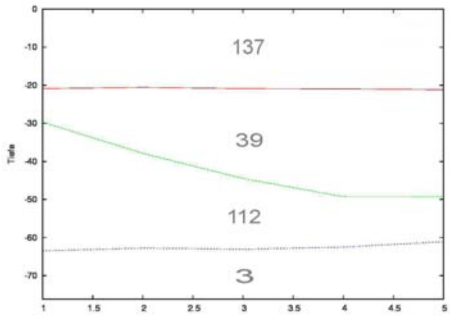

Figure 6: LCI with costraints of adjacent resistivities, resistivity values ρ_i in Ωm. $\chi = 0.93$; 6 iterations.

lems like this. In Fig. 7 and 8 a fit is shown for polynomials of the order 5 and 12. The synthetic data has no errors. Comparing the results, the course of the nonlinear boundary is already fitted well by the polinomial of order 5. In Fig. 8 appears an extrem misfit at the borders. This happens, because the higher the order of the polynomials, the more unknown the inversion has to determine, compared to the length of the measurement lines. It turns out, that orders from 4 to 7 are acceptable, where higher orders do not improve the results. Inverting the model from Fig. 4 with a series expan-

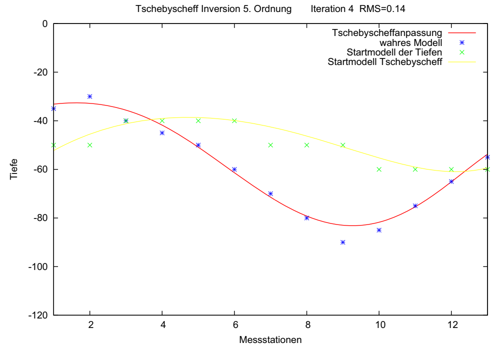

Figure 7: Chebyshev fit with polinomials of order 5 (red) for a 2 layer boundary (blue), initial depth (green) and initial polynomial (yellow).

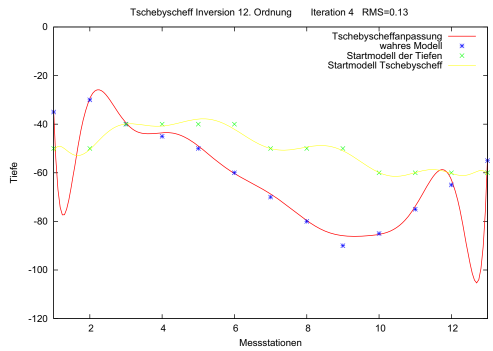

Figure 8: Chebyshev fit with polinomials of order 12 (red) for a 2 layer boundary (blue), initial depth (green) and initial polynomial (yellow).

sion of order 5 does not succeed, because the misfit creeps without appreciable progress over the iterations and does not converge. A reason is, that the chebyshev inversion has more parameters to determine (18 coefficents and 20 resistivities) than the LCI (15 thicknesses and 20 resistivities) for the same data information. Unfortunately the parameter dependencies for the chebyshev inversion are also more difficult to solve. An approach for this problem is to enhance the

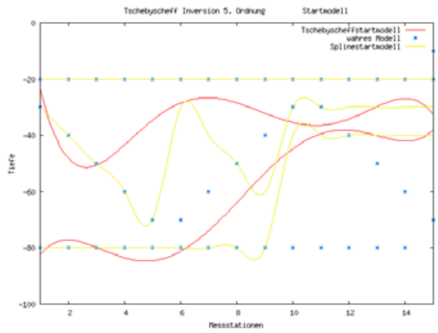

Figure 9: Chebyshev fit with polinomials of order 5 (red) reference model (blue), initial depth (yellow) described by a cubic spline.

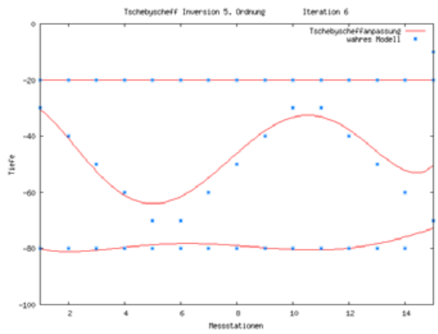

Figure 10: Chebyshev fit with polinomials of order 5 (red) reference model (blue). 6 iterations.

line of measurements or decrease the order of the polynomials. Fig. 9 and 10 show a result for a four layer model with fifteen sites, and no data error. In the initial model the resistivities are kept the same as in the reference model, only the depth are varied (Fig. 9). This shows, that the chebyshev inversion needs much more data information to achieve satisfying results.

2D synthetic data

The data for 2D models was generated with the program sldmem3t, that was written by

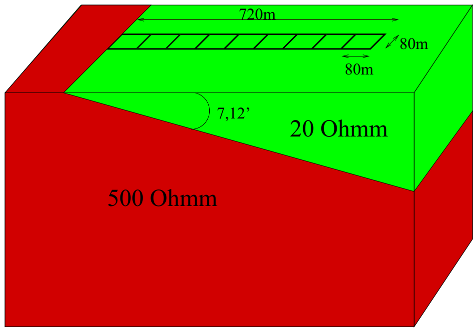

Figure 11: 2D Model with dipping layer and 9 central loop sites.

Druskin and Knizhnerman (1988) and base on finite differences. To test the ability of the inversions to compensate the slightly different transients for a 2D case with the regularisations based on a 1D forward calculation the data for a two layer subsurface with dipping layer of $7,2°$ was calculated (Fig. 11). The data was inverted without constraints (Fig. 13) and with constraints on adjacent resistivities (Fig. 14). The initial model is shown in Fig. 12. The inversion was stoped, when χ could not improve anymore. The data misfit is 1% gaussian distributed, increasing for late times.
The inversion without constrains is not capable to resolve the reference model on site 6 and 7 (iteration 6; $\chi = 20.2$), more iterations does not improve this result and the resistivity and depth values become even worse, whereas the inversion with lateral constraints retrieve the values of the reference model well (iteration 5; $\chi = 1.87$.). Comparing this two inversions the lateral constraints have an clear advantage here, because the outliers on site 6 and 7 can be balanced by the constraints. In Fig. 15 the results of the chebyshev inversion can be seen. They are almost as good, as the LCI

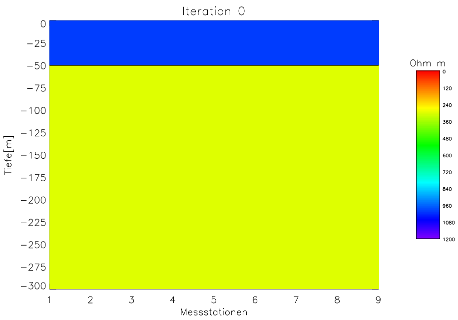

Figure 12: Initinal model

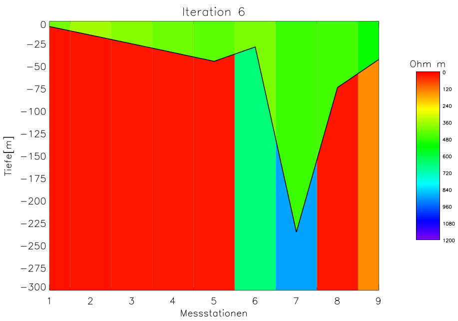

Figure 13: Inversion without constraints, $\chi = 20.2$

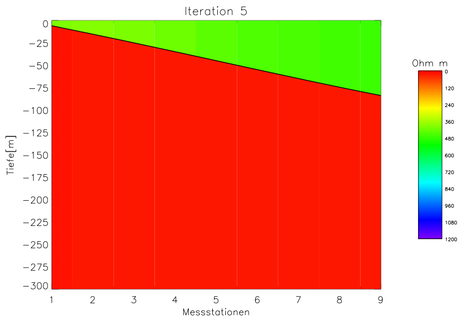

Figure 14: Inversion with lateral constraints, $\chi = 1.87$

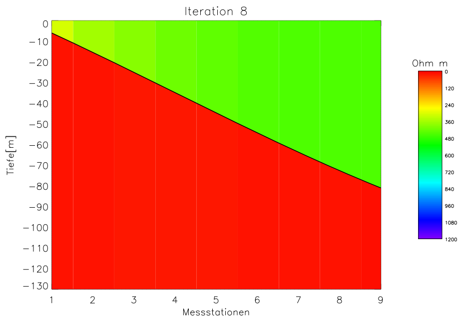

Figure 15: Chebyshev inversion, $\chi = 1.72$

results (iteration 9; $\chi = 1.72.$) , but the resistivities on site 1 are slightly too low and the chebyshev inversion needed 9 iteration for this result.

Conclusion

Assuming a quasi-layered subsurface, both methods are capable of achieving results from 1D and 2D synthetic data. LCI has proved as a reliable tool, that has short calculation times. The results are in genenal preferable to an inversion of a profile without constraints.

The Chebyshev inversion needs large profiles for a convergence on the misfit. The numerical efford is more expensive and therefore the calculation time is larger than for the LCI, but both methods need just fractions of time compared to a full 2D inversion. The advantages of the chebyshev inversion should futher be tested for long measurement lines.

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