On Multisheet and Volume-3D Modelling of the Baltic Shield

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1. Introduction

During the BEAR experiment (*Baltic Electromagnetic Array Research*) about 50 magnetotelluric (circles) and 20 magnetovariational stations (triangles), shown in **figure 1**, operated simultaneously on the Baltic Shield for 6 weeks with a sampling rate of 2 seconds. This long period array experiment with an average site spacing of 150 km aims at resolving the conductivity structure of the lithosphere-asthenosphere system through 'resistive windows' of this ancient shield. However, distortions due to conductivity anomalies and source effects cause serious problems. 3D forward modelling, based on a-priori information on crustal conductances and external equivalent current systems, is an excellent tool to study both types of distortion on electromagnetic responses. Two techniques were applied to model the Baltic Shield: *i)* a quasi-3D *multisheet* model with a fine horizontal grid cell resolution and *ii)* a *volume-3D* model with a fine vertical discretisation of the crust. This paper briefly compares both modelling techniques and shortly demonstrates applications of these studies in comparison with experimental data, i.e., the investigation of plane wave response functions, source effect studies based on a polar electrojet model, and testing the presence of a hypothetical asthenosphere.

2. Electromagnetic multisheet and volume-3D modelling

Two modelling techniques were used: a quasi-3D approach applying the thin sheet technique to several layers (multisheet modelling) and a volume-3D modelling scheme. The corresponding codes ('XPLATE' and 'X3D') are by Avdeev, Kuvshinov and Pankratov (e.g. Avdeev et al., 2002), basing on the integral equation technique and combining Krylov subspace iteration with the modified iterative dissipative method. In addition to various tests, which show for example excellent agreement with synthetic COMMEMI models, comparisons with finite difference codes applied to the very complex model of the Baltic Shield (introduced below) yielded minor deviations of electromagnetic fields in the order of a few percent only (Varentsov et al., 2002).

Both modelling tools are sketched in **figure 2**. Either plane wave excitation in two polarisations or an inhomogeneous source was applied; a schematic polar electrojet illustrates the latter. The earth model has cells of heterogeneous crustal conductivities that are embedded in a normal structure (1D):

i) The *multisheet model* describes crustal heterogeneities by three inhomogeneous thin sheets representing upper, middle and lower crust, embedded in a layered normal structure. Each sheet consists of 360 x 336 quadratic model cells of 10 km in base length.
ii) The *volume-3D model* consists of 9 inhomogeneous layers with varying thicknesses between 2 km and 10 km, representing the whole crust down to 60 km depth. Below, a layered normal structure represents the mantle. Each layer consists of 180 x 168 quadratic model cells of 20 km in base length.

Both approaches are comparable in their numerical expense and limitations (e.g., memory or computation time) and appropriate for a conventional PC. The advantage of a fine horizontal discretisation by multisheet modelling is achieved by a simplified vertical structure, neglecting the vertical dimension of crustal heterogeneous sheets (thin sheet approximation, justified only for periods associated with mantle penetration depths). The advantage of a fine vertical discretisation by volume-3D modelling is achieved by a less resolving horizontal discretisation. However, the normal structure below crustal depth coincides in both approaches. The huge anomalous modelling area (3000 km x 3360 km, extending 10 times the BEAR array area) guarantees that the surrounding homogeneous normal structure, which is required by the integral equation formulation, does not effect model responses within the BEAR array. Engels et al. (2002) describe more details of multisheet modelling and their results.

3. Crustal a-priori model

The conductivity structure of the earth model was derived from a conductance map in digital form including Fennoscandia and surrounding areas. This database, compiled by Korja et al. (2002; contains a comprehensive list of references), consists of various inhomogeneous layers:

- *Water conductance*: The conductance of surrounding oceans and the Baltic Sea is based on sea water conductivity and bathymetric data.
- *Sediment conductance*: Thicknesses of sea and continental sediments were converted into conductances, using conductivity information mainly from magnetotelluric data.
- *Basement conductance*: Six layers, each representing a 10 km thick slice of crust, describe the whole crust down to the crust-mantle boundary. Conductances are based largely on results from hundreds of magnetotelluric and magnetometer array sites, which operated shield-wide during the past two decades. Airborne magnetic data and other geophysical and geological data were used as indication for identifying units of different resistivities.

Furthermore, collecting normal models (1D) for various parts of the shield helped to estimate the normal background structure used here in modelling. The three thin sheets of multisheet modelling are derived directly by superimposing corresponding conductance layers; the nine layers of volume modelling are obtained by converting conductance layers into resistivities. The results of BEAR data did not enter the database yet. An upgrade, corrections from BEAR data and extensions of the database are intended at a later stage - thus, actual modelling is based purely on an a-priori earth model.

The conductance map of the upper crust (0-10 km depth) is given in **figure 3**, including conductances of seawater and sediments, while **figure 4** represents the middle crust (10-30 km depth). Model cells of 10 km base length, referring to multisheet modelling, are indicated by tick marks on axes; the total model area is even enlarged by further 600 km in all directions. In general, the very resistive Baltic shield (conductances of partly 1 S, integrated over the whole crust) contains strong anomalies (up to several 10,000 S) and is surrounded by conductive seawater and sediments. Anomalies in the middle crust are often shaped as conductive belts, which may trace former tectonic plate boundaries. Archaean Karelia is the most resistive part of the shield.

4. Response function

A qualitative comparison between multisheet and volume-3D modelling is illustrated in the following four figures for apparent resistivity and impedance phase, both referring to the determinant of the impedance tensor. The modelling period here is 2048 seconds, as in all following figures. Apparent resistivities revealed from multisheet modelling, e.g. **figure 5**, reflect widely the distribution of crustal conductances (figures 3 and 4). Even though this long modelling period is associated with mantle penetration depth, the screening effect by crustal anomalies is clearly visible. The typical apparent resistivity behaviour of B-polarisation at large conductance contrasts ('oscillation') is still visible in determinant mode, e.g., the coast effect along the Norwegian coastline. The latter and its influence reaching deep into the resistive Baltic Shield is clearly imaged by induction arrows (Engels et al., 2002). The corresponding effective phase in **figure 6** covers a wide range, extending from a few degrees in areas of deep seawater to about 70° in very resistive parts of the shield. A few model cells (left white) indicate even out of phase behaviour at very complex 3D conductivity structures.

For volume-3D modelling apparent resistivity and impedance phase in determinant mode is shown in **figure 7** and **figure 8**, respectively. At first glance, the general behaviour of these response functions compared with multisheet results (figures 5 and 6) is quite similar, but differences show up in details. Values of apparent resistivity and phase vary less extreme in the volume-3D model and tend towards more 'realistic' values. Effective phases do not leave the quadrant any more, thus local conductivity extremes in the earth model were reduced significantly. Note that some impedance tensor elements Z_{xy} and Z_{yx} (not determinant) still have data out of phase in the vicinity of complex 3D structures.

The thin sheet model geometry, i.e., partly very conductive sheets embedded in a very resistive - almost isolating - normal structure belonging to the shield, causes high contrasts in conductance. Induced currents often are highly concentrated in these thin sheets. The current density distribution within the sheets (Engels et al. 2002) images strong current channelling at depths along elongated bands of conductors (e.g. figure 4), which might be exaggerated by the geometry of the thin sheet model. At those large conductances, the thin sheet condition starts to become problematic. In contrast, the rougher horizontal discretisation by volume modelling (1 cell with 20 km in base length instead of 4 cells with 10 km in base length) has an averaging effect and reduces contrasts. Thus, volume modelling seems to be preferable due to a more realistic crustal model in vertical scale in spite of a lower resolution in horizontal scale.

However, differences in response functions at most of the BEAR sites are often small. Some examples as a function of period are given in **figure 9**. Multisheet (solid line) and volume-3D (asterisks) model responses are compared with experimental data (crosses) for selected BEAR sites in different shield areas. Presented transfer functions of experimental BEAR data are an average of various processing scheme results applied by different groups, which show in general a fairly well agreement (except for very long periods). For exceptional sites where conductance contrasts are extreme, e.g. site B29, multisheet modelling becomes numerically doubtful (thin sheet approximation violated), while volume-3D model responses still confirm measured data. Note that experimental data are effected by static shift and that modelled responses refer to the a-priori model. A real 'fit' should not be expected, but a first order agreement is achieved for most of the BEAR sites. Owing to source effects, phases of experimental long period data larger than 3,000 seconds partly behave inconsistent.

5. Source effects

For a simplified prototype model of a polar electrojet, **figure 10** shows the corresponding equivalent current system. The model approximates current densities by a dense grid of electrical dipoles at ionosphere heights. This sheet current system produces identical electromagnetic responses on ground as a loop-like 3D current system, connecting the horizontal jet current in the ionosphere by vertical field aligned currents, as sketched in figure 2. Instead of field aligned currents in the 3D current system, return currents show up in the equivalent current model. For induction purposes, the question about the physically real current system (a contribution of both) is irrelevant. In this model, current densities vary harmonically in time, but remain static in space. Source effects in terms of apparent resistivity and impedance phase (determinant mode) are presented. In order to obtain two polarisations for resolving the full impedance tensor, the jet excitation has been superimposed to one (out of two) plane wave excitation, choosing an amplitude factor of 20 between jet and plane wave background field for the surface magnetic field below the jet centre. In analogy to real data processing, plane wave events might superimpose with events of inhomogeneous sources.

The electrojet model of figure 10 has first been applied to the normal structure - a 1D earth model. Below the jet centre, negative values (red) in **figure 11** indicate apparent resistivities partly underestimated by 50%. Beyond the jet current, the positive (blue) area indicating overestimation is accompanied by a ring structure of moderate underestimation (increasing in intensity with period!). The latter ring structure is due to the source geometry, the equivalent or 3D loop-like external current system. Note that a band or line current model extending from infinity to infinity does not produce such a ring structure of underestimation - it is due to the finite extension of the jet current only.

In **figure 12**, the Fennoscandian 3D earth model has replaced the 1D earth model. Now, the distortion pattern of apparent resistivity differs considerably from the previous response reflecting purely the source geometry. Areas of under- and overestimation cannot be predicted any more by a geometric function - distortion is not any more purely a function of relative position with respect to the jet location. Instead, the internal 3D conductivity structure strongly modifies the distortion pattern and is essential for understanding source effects. Furthermore, distortion depends strongly on frequency - areas of underestimation might turn to overestimation and vice versa.

Source effects in effective phase for the 3D earth model, see **figure 13,** shall be compared with the corresponding plane wave response in figure 8. An increase of effective phase - and even out of phase (areas left white) - is observed in the resistive shield south of the jet current (but less pronounced in the conductive ocean north of the jet current). This is a typical sign for source effects. However, a decrease of phases can also be observed at a few other locations. Experimental data at long periods partly show such a considerable increase in phase - often not in accordance with the apparent resistivity behaviour. Substorm electrojets, which occur occasionally at irregular times, might be removed by robust codes. But in case of a systematic distortion ('bias') owing to quasi-periodic convection electrojets, which are present in most long period events, robust processing schemes applied to BEAR data have to fail and source effects remain.

6. Asthenosphere

In order to test the hypothesis of a well conductive upper mantle ('asthenosphere'), as often indicated by experimental data, residuals in effective phase (experimental minus modelled phase) have been compared for two different reference models. The standard normal model of the Baltic Shield has a gradually increase of conductivity with depth and consists of a layer of 100 Ωm between 200-300 km (1000 S). This layer has been replaced in the alternative 'asthenosphere' model by a more conductive layer of 20 Ωm (5000 S).

For the standard model, **figure 14** illustrates for a period of 2048 s the difference between experimental and modelled phase. Residuals are clearly biased towards positive values (red). This indicates too low values of modelled phases or, in other words, too small conductances at upper mantle depth. For the alternative 'asthenosphere' model with increased conductance at depths between 200-300 km, residuals are shown in **figure 15.** The previously seen bias between observed and modelled phase data in figure 14 is removed – phase values scatter around zero (or even around a slightly negative mean value). Thus, modelling supports the existence of a good conductive upper mantle. A similar comparison of apparent resistivities supports this observation, but is less convincing due to unknown static shift effects in the data.

7. Concluding Remarks

Multisheet and volume-3D modelling of a large shield area are possible owing to powerful modelling codes and detailed a-priori conductance information. Multisheet modelling seems partly to increase extremes in electromagnetic responses due to the geometry of the thin sheet earth model.

Various responses and invariants have been studied for a period range from 100 to 100,000 seconds (only a selection is shown here) and show a first order agreement with experimental data. Source effects were modelled by means of equivalent current systems for a 3D source and a 3D earth model. Distortions turn out to behave in a nearly unpredictable way, depending not only on source geometry, but also on the internal conductivity structure and frequencies investigated. A well conducting upper mantle is indicated by comparison between experimental and modelled impedance phases.

Several further applications of 3D forward modelling, in comparison to analysis of experimental BEAR data, were undertaken in collaboration with the BEAR Working group (in preparation for publication), i.e.,

- Distortion and dimensionality analysis: How convincingly do decomposition and dimensionality parameters work for a complex synthetic 3D model?
- 1D/2D inversion tests for resolving the asthenosphere: Can the asthenosphere be resolved in presence of strong crustal anomalies hosted within a resistive shield?
- Structural 'anisotropy' versus intrinsic anisotropy for central Finland: How much of the phase split along the SVEKA profile is reduced to surrounding isotropic conductivity anomalies and how much is left to intrinsic anisotropy beneath the profile?
- Testing the HSG (horizontal spatial gradient) method by modelling inhomogeneous sources in presence of crustal distortion: Is the gradient method able to provide a geomagnetic long period response function for resolving the deep normal structure?

8. Acknowledgements

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9. References

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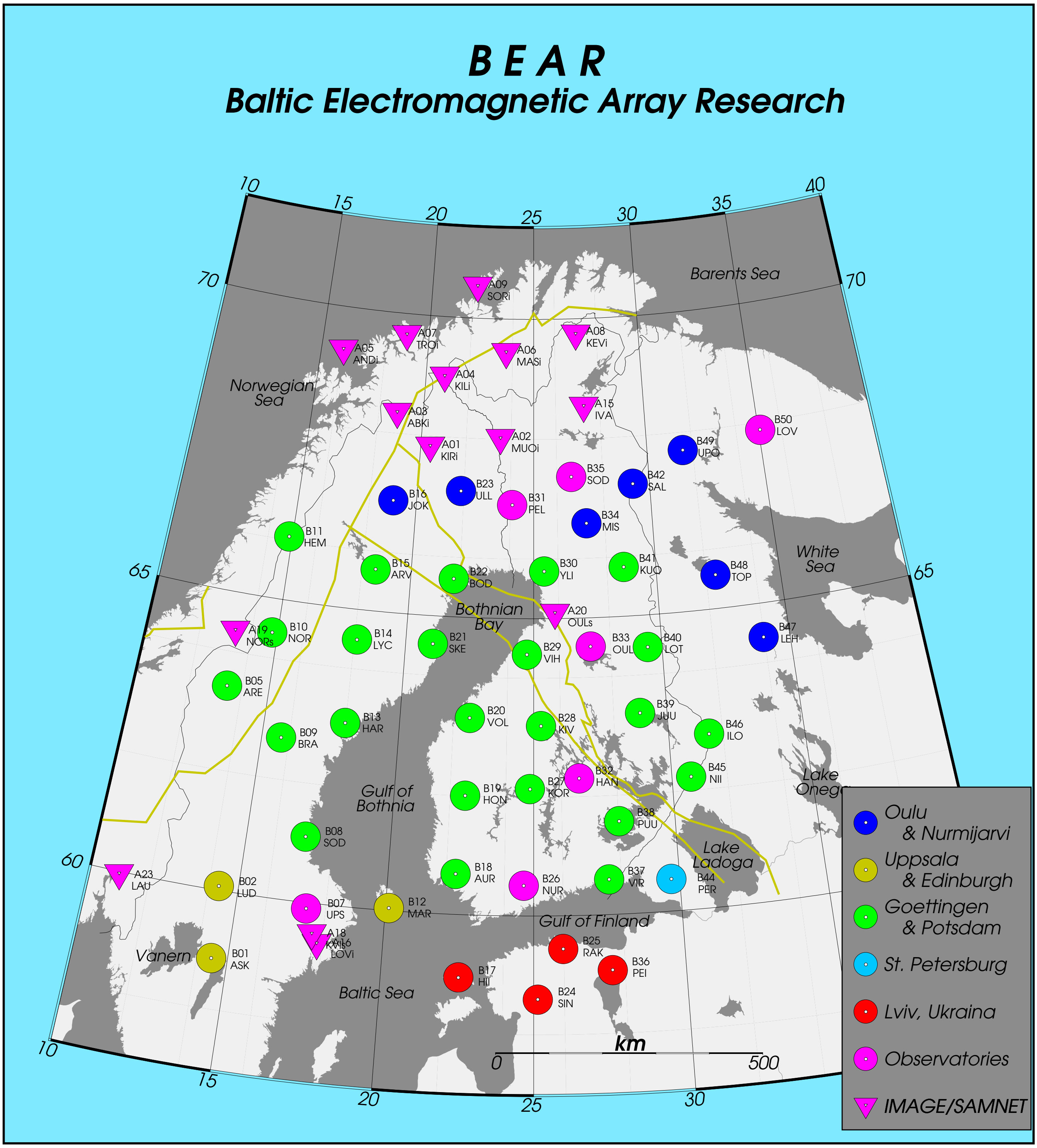

Figure 1: *BEAR array. Circles indicate magnetotelluric and triangles magnetovariational stations.*

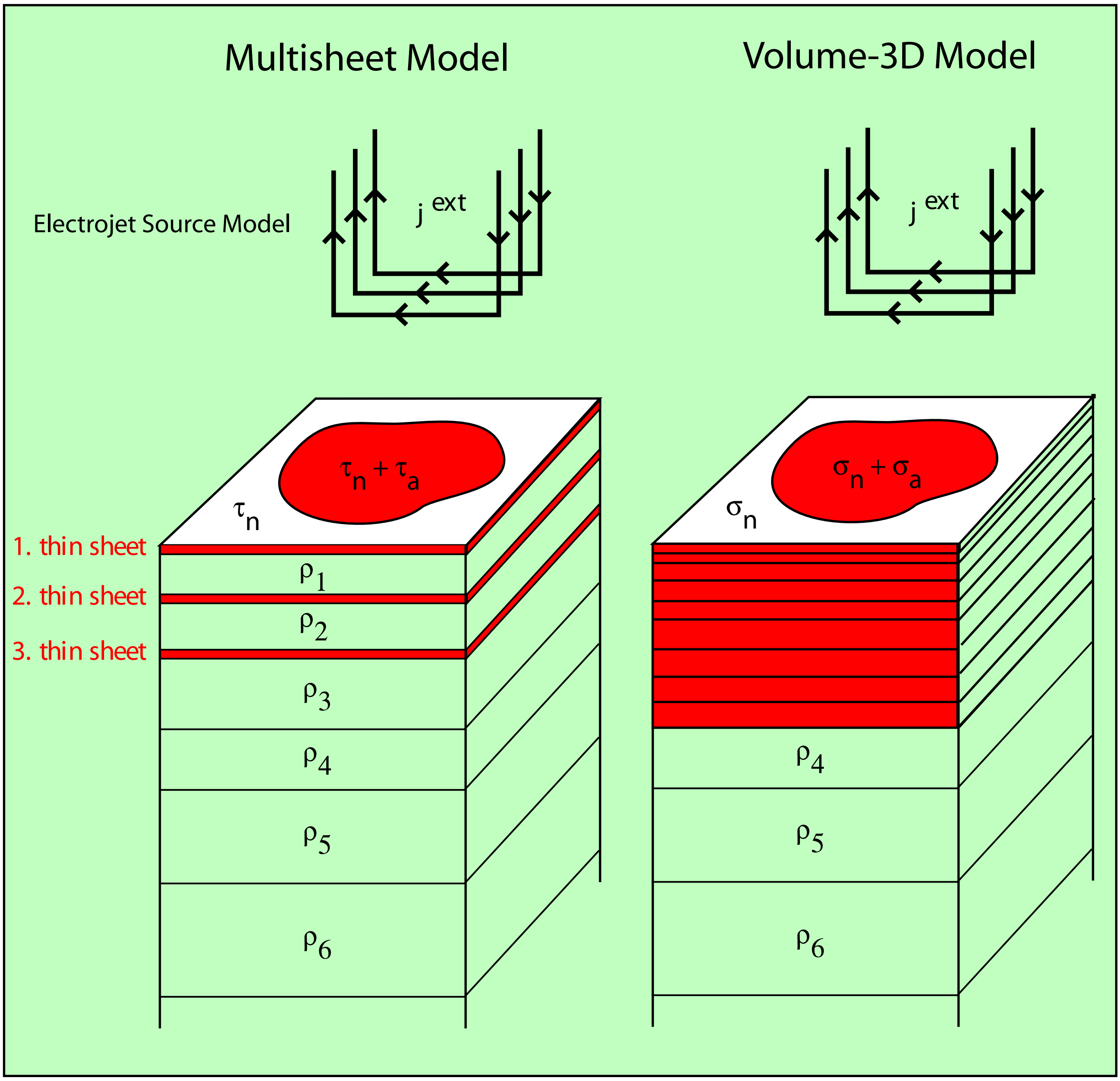

Figure 2: *Multisheet model versus volume-3D model.*

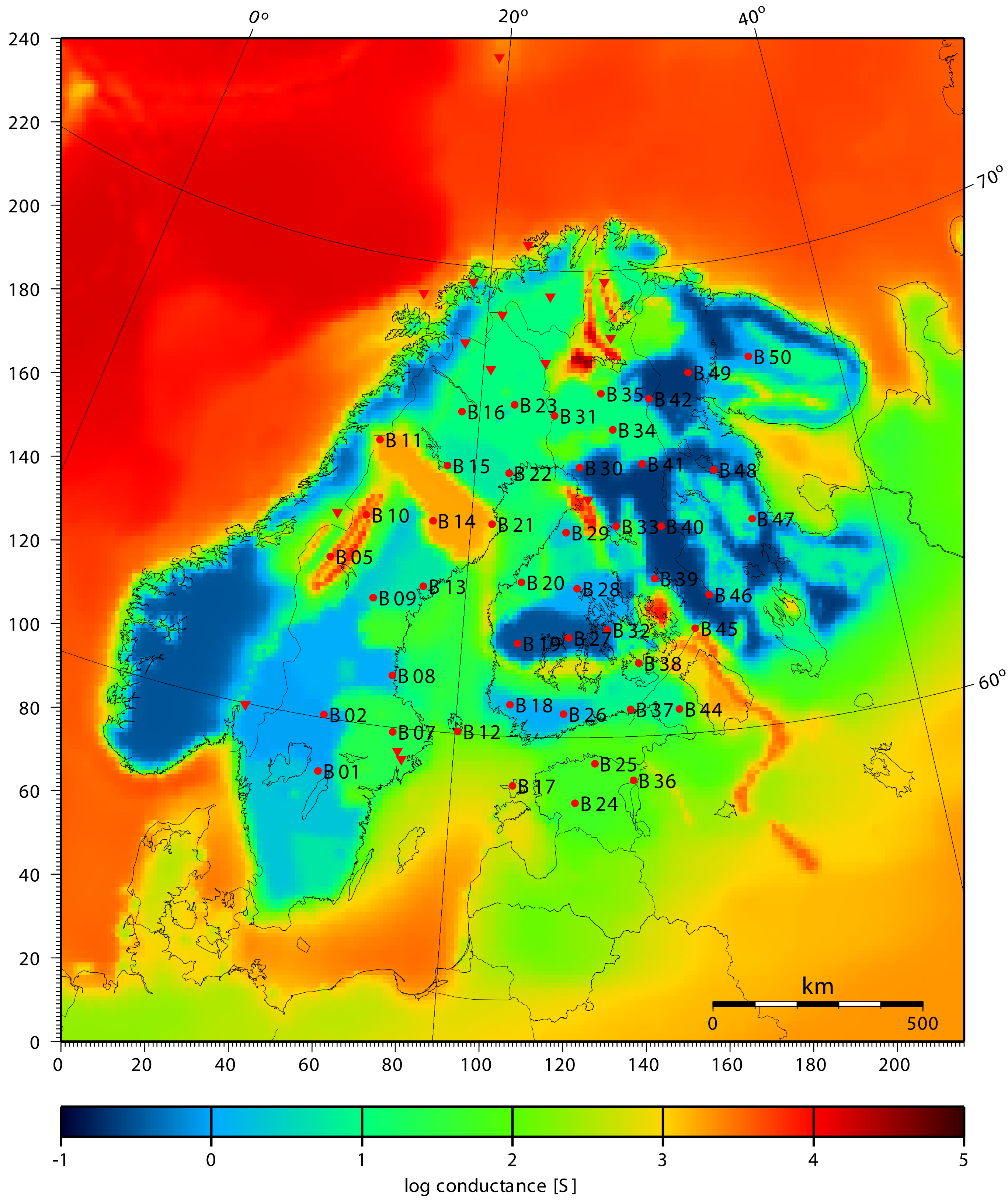

Figure 3: *Conductance map of the upper crust (0-10 km) including seawater and sediments.*

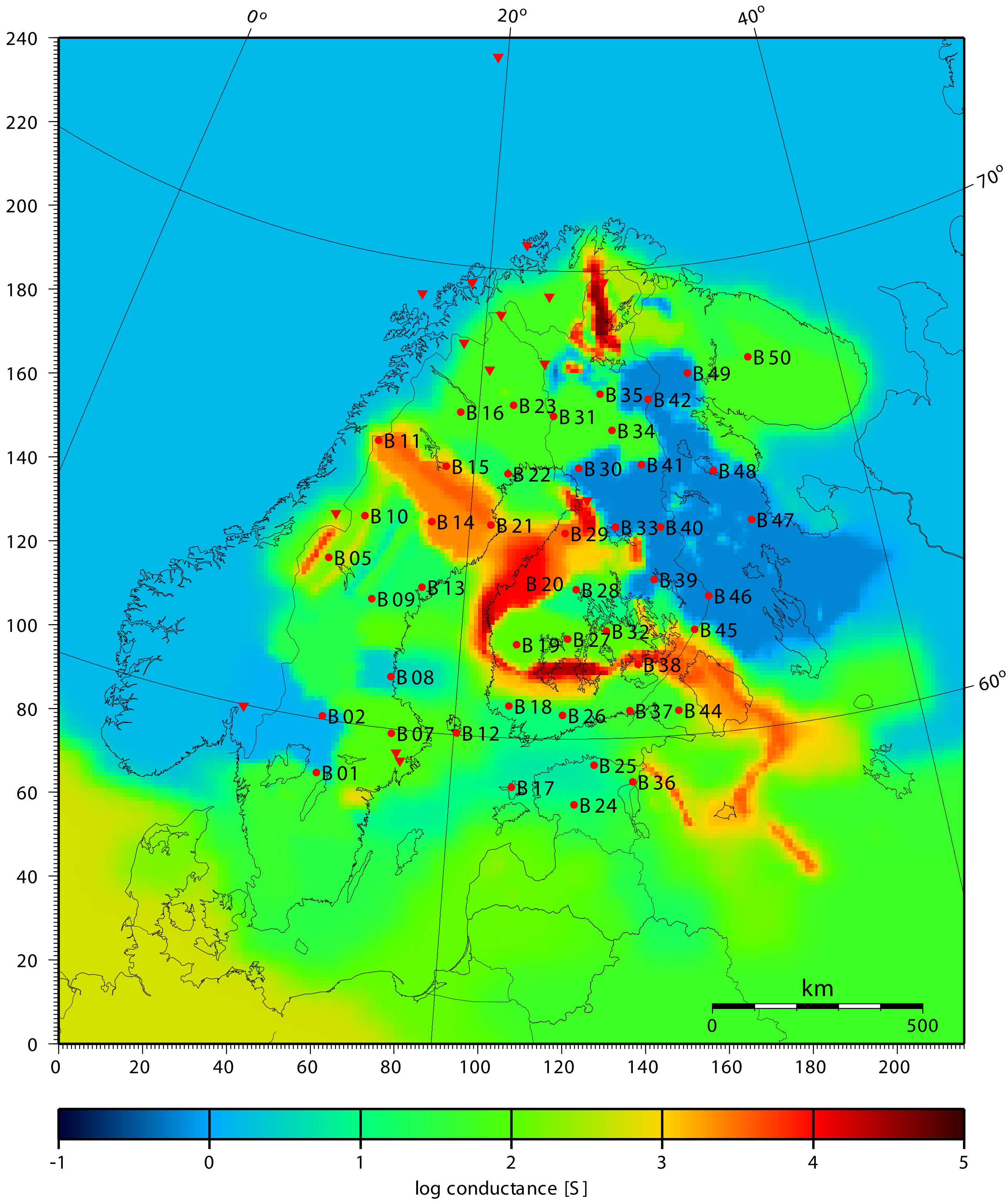

Figure 4: *Conductance map of the middle crust (10-30 km).*

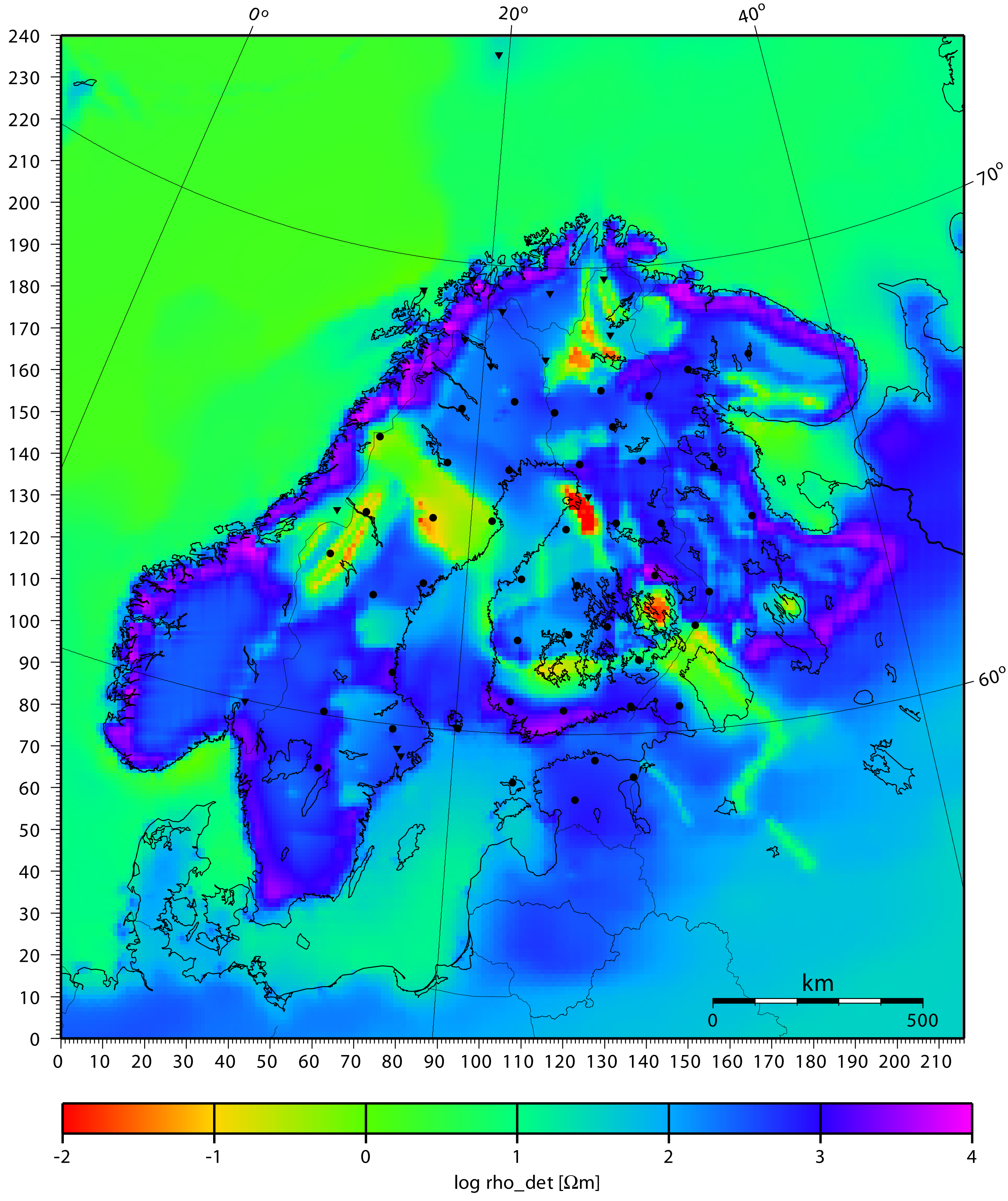

Figure 5: *Multisheet Model: Effective apparent resistivity (T=2048 s).*

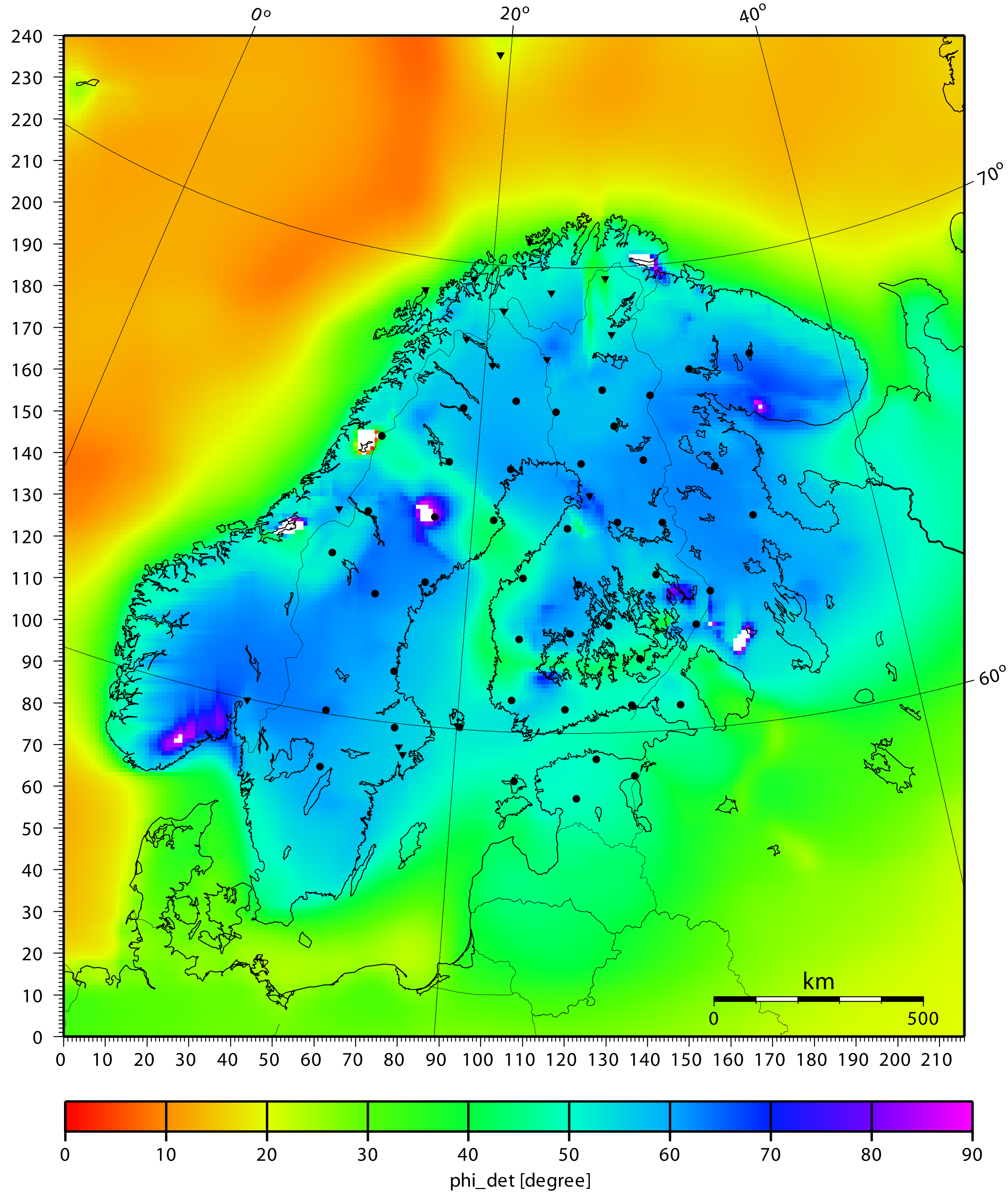

Figure 6: *Multisheet Model: Effective impedance phase (T=2048 s).*

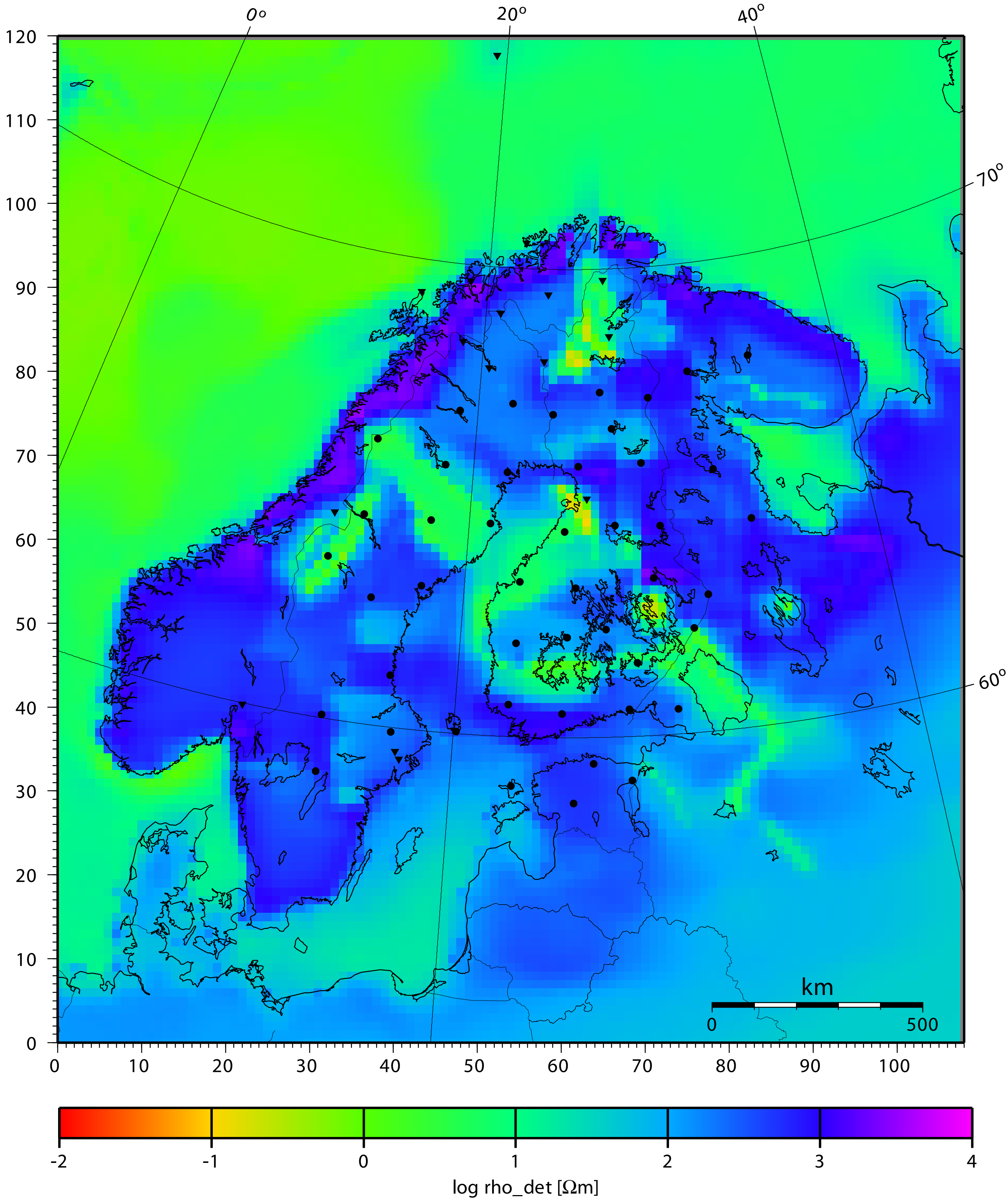

Figure 7: *Volume-3D Model: Effective apparent resistivity (T=2048 s).*

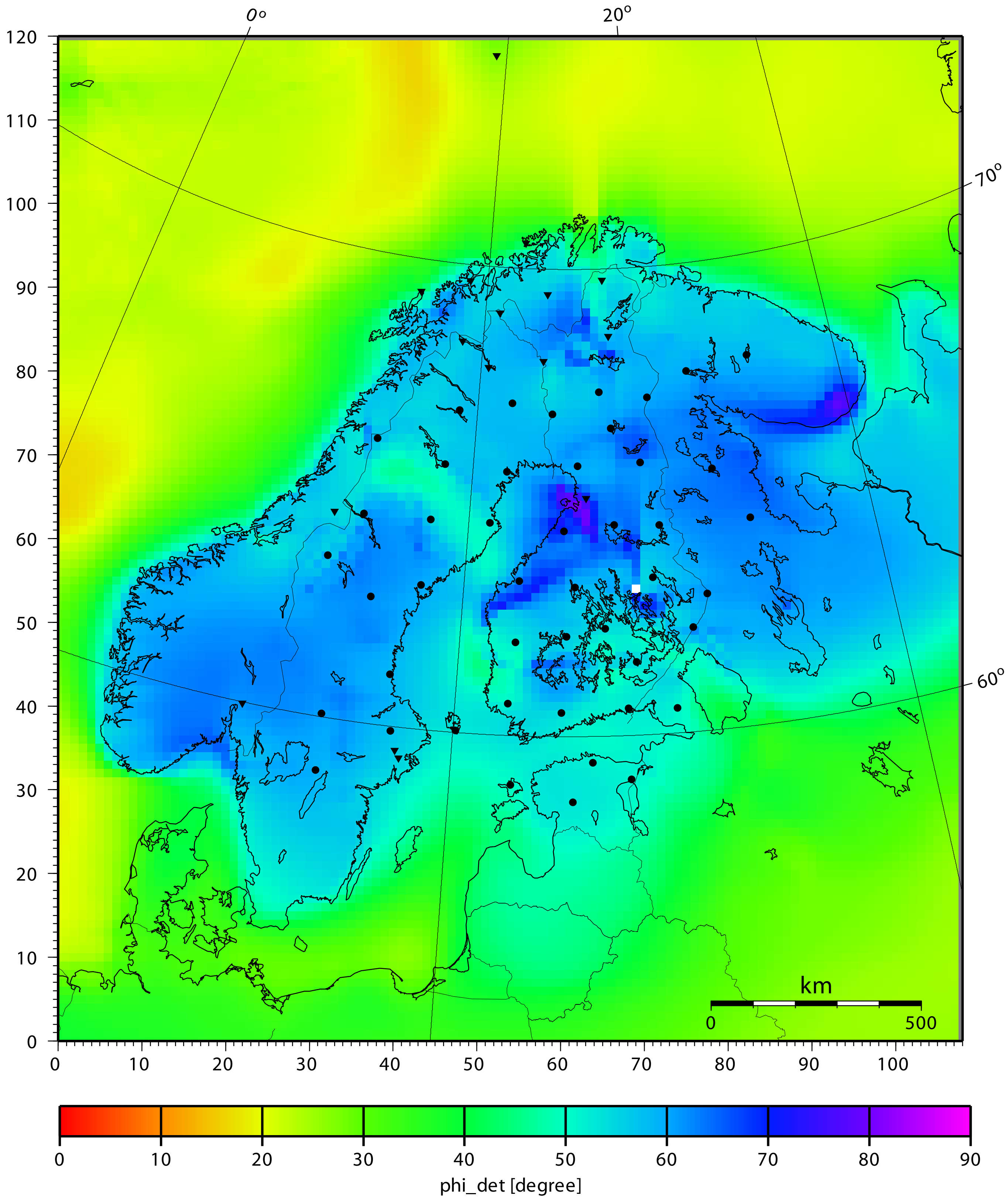

Figure 8: *Volume-3D Model: Effective impedance phase (T=2048 s).*

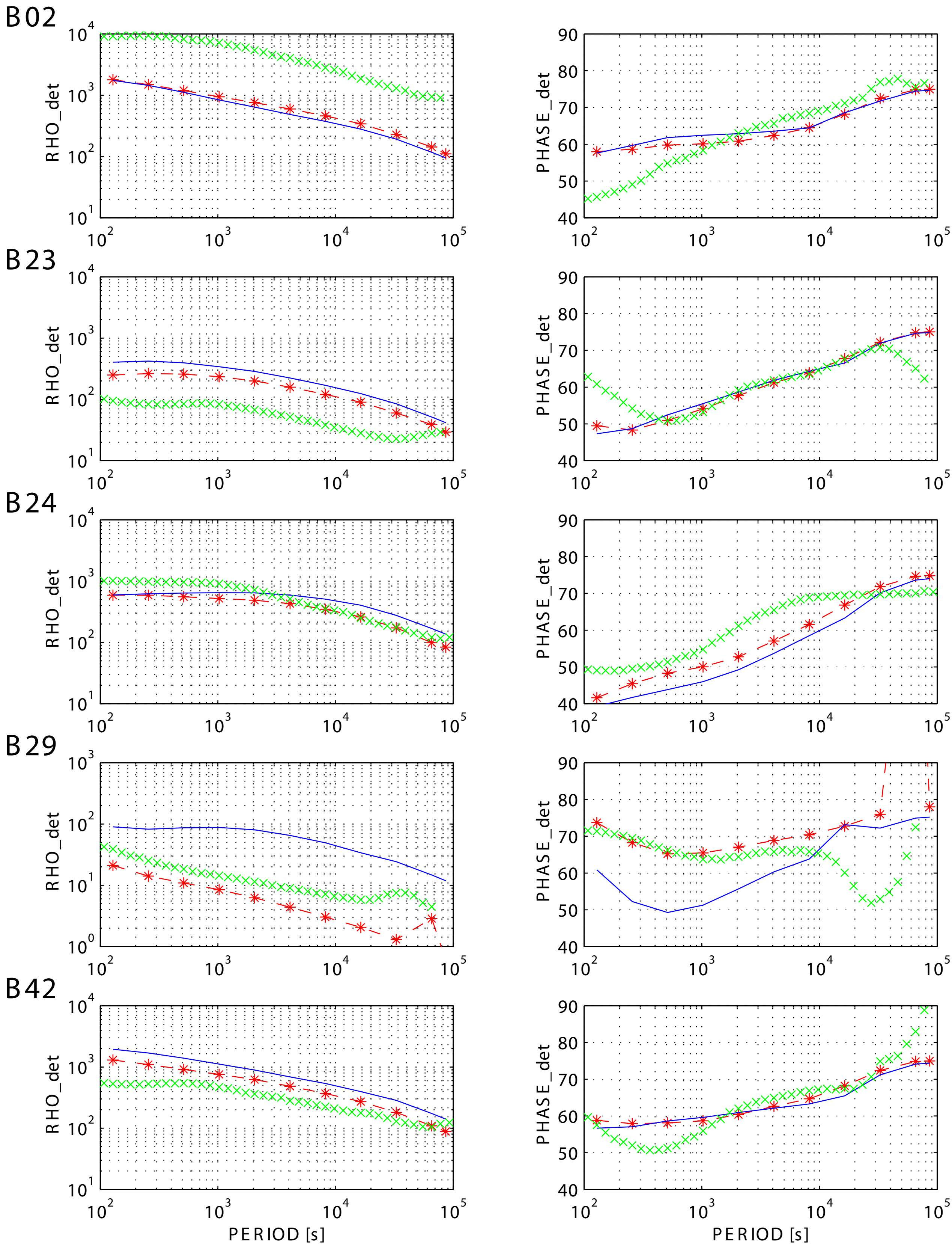

Figure 9: *Effective apparent resistivity (RHO_det in Ωm) and impedance phase (PHASE_det in degree): BEAR experimental data (green), multisheet modelling (blue) and volume-3D modelling (red) responses.*

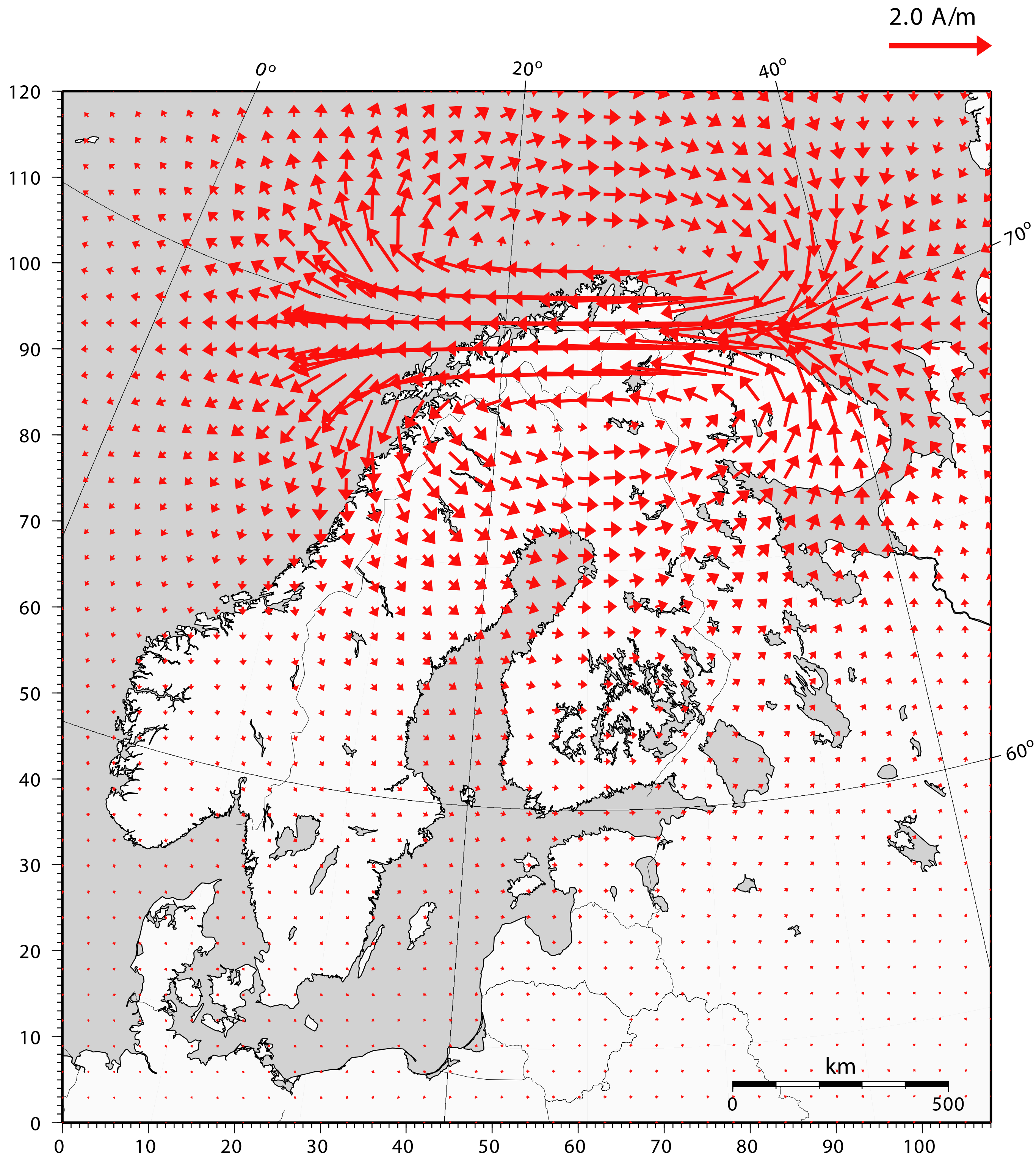

Figure 10: *Polar Electrojet Model: Sheet current densities of an equivalent current system.*

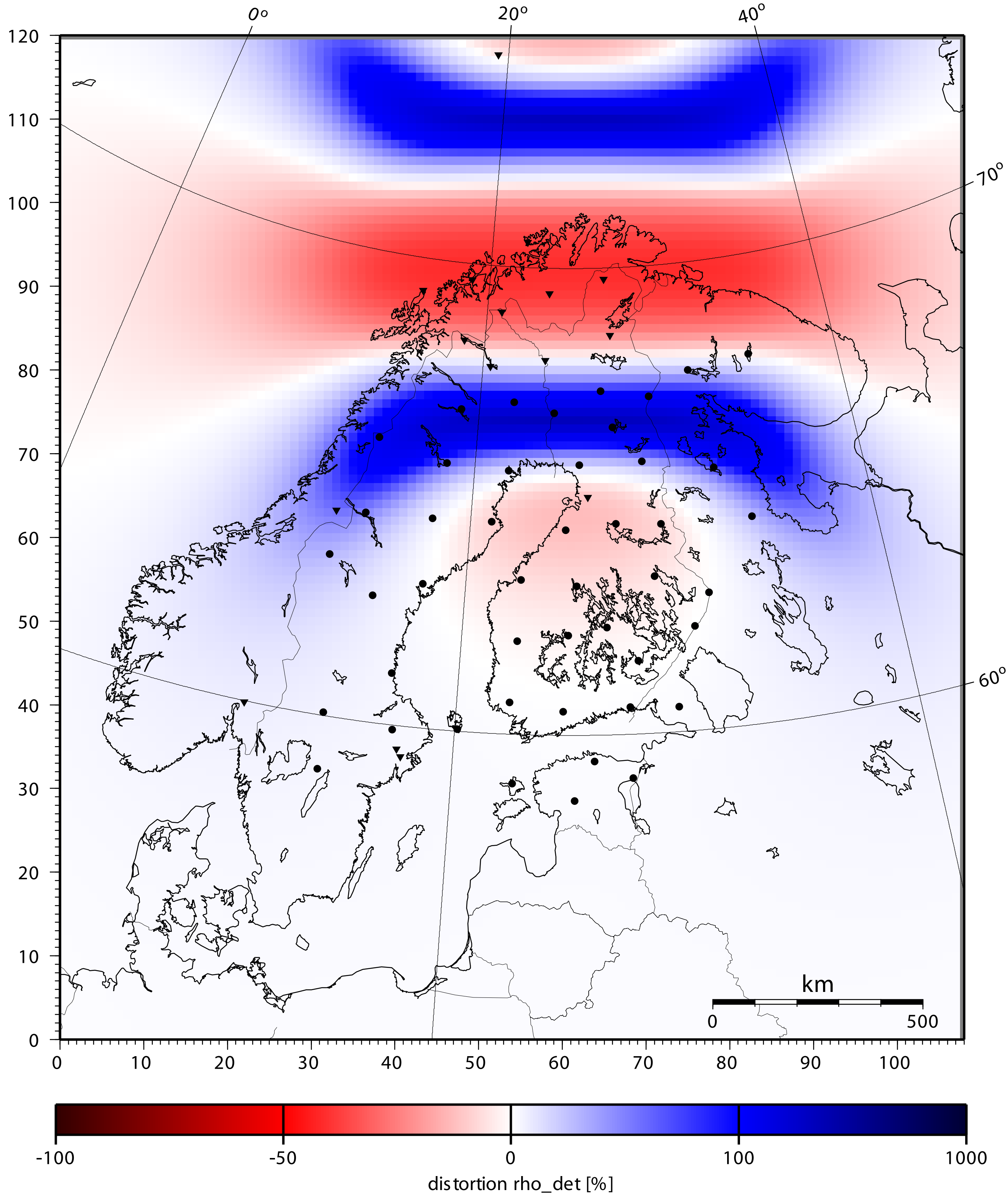

Figure 11: *Source Effects for 1D Earth Model: Distortion of effective apparent resistivity (Electrojet minus plane wave response)*

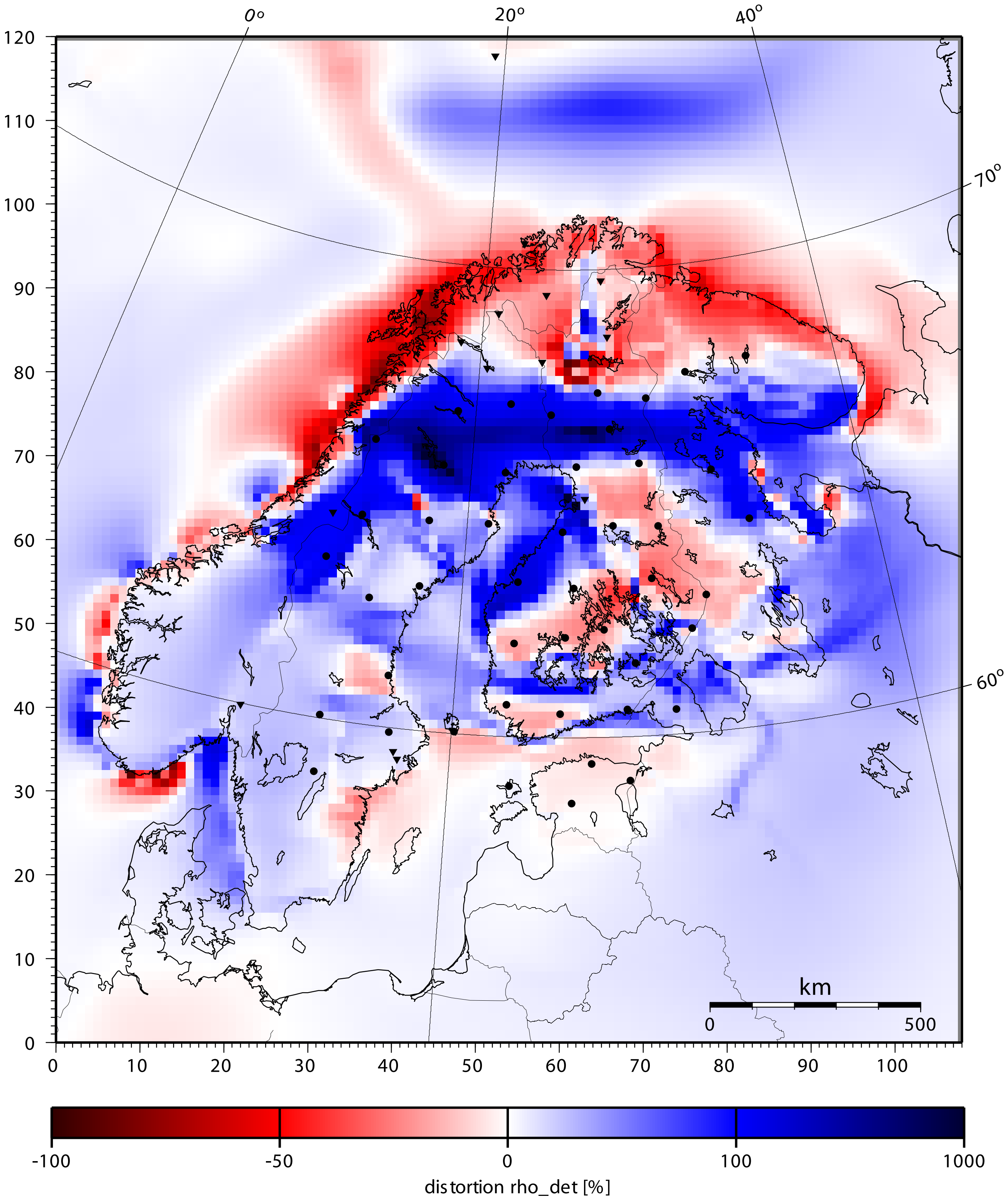

Figure 12: *Source Effects for 3D Earth Model: Distortion of effective apparent resistivity (Electrojet minus plane wave response).*

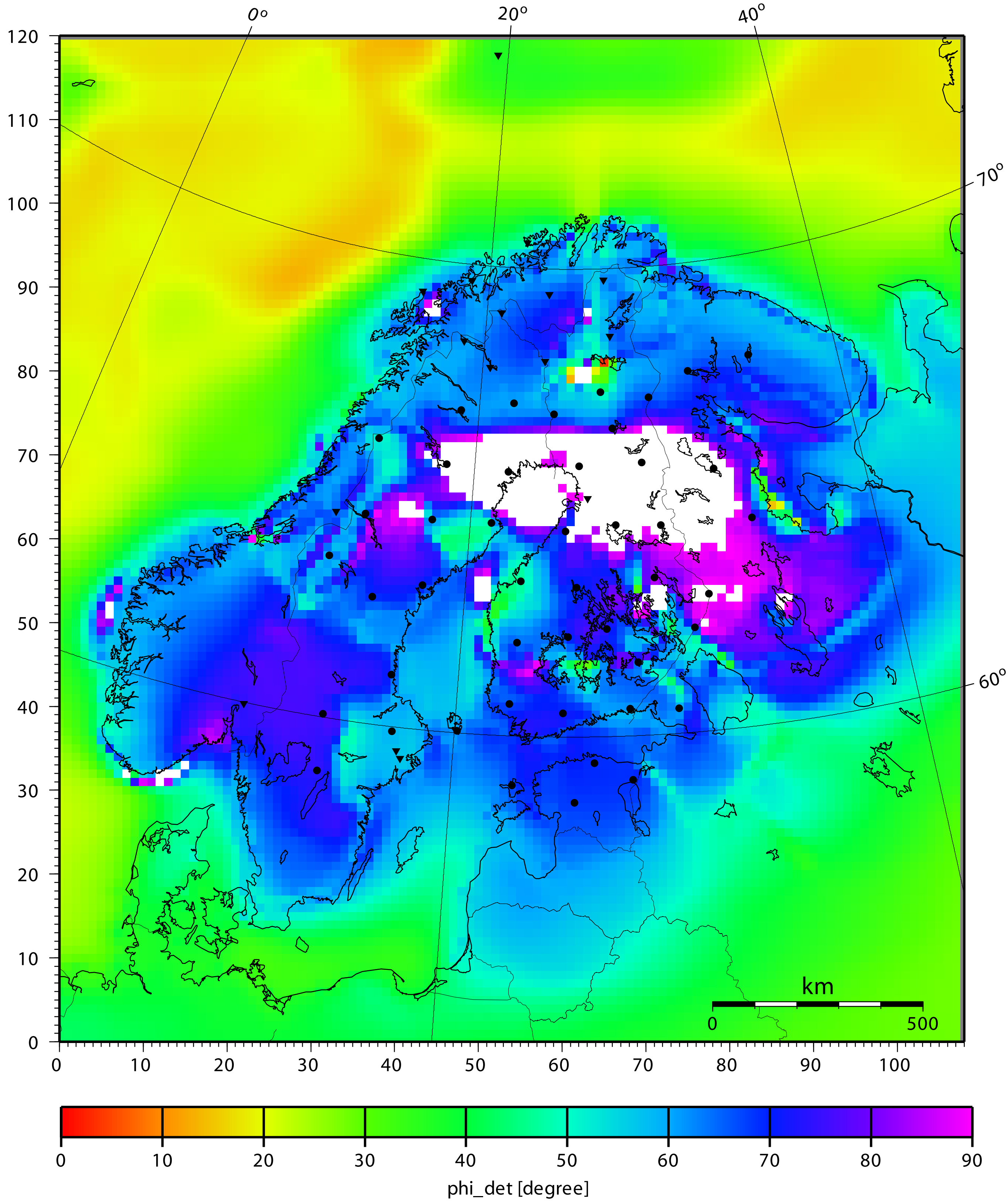

Figure 13: *Polar Electrojet and 3D Earth Model: Effective impedance phase phi_det (T=2048 s). White model cells are out of phase (exceeding 90 degrees).*

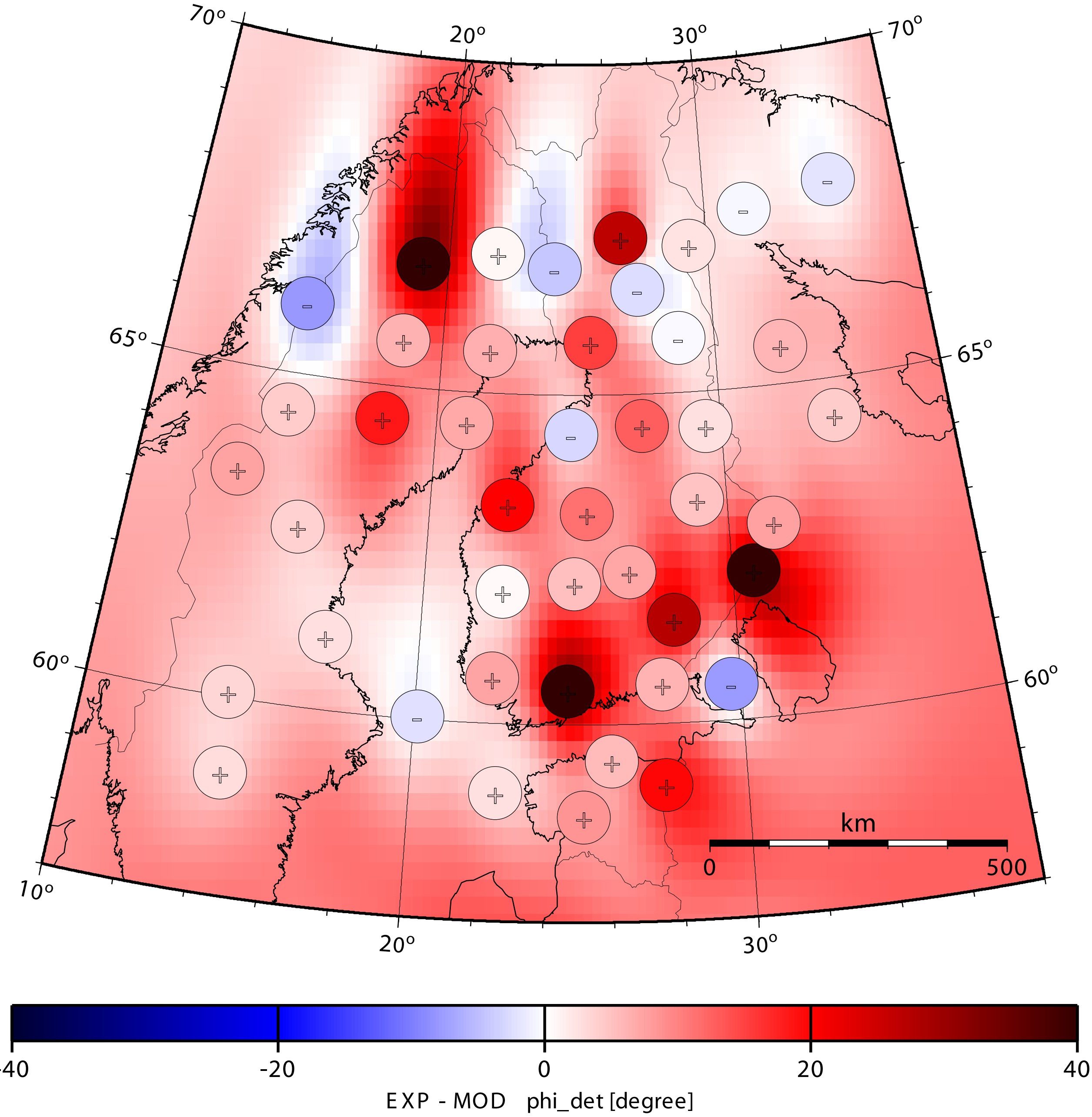

Figure 14: *Residuals in effective phase: Experiment minus model without asthenosphere.*

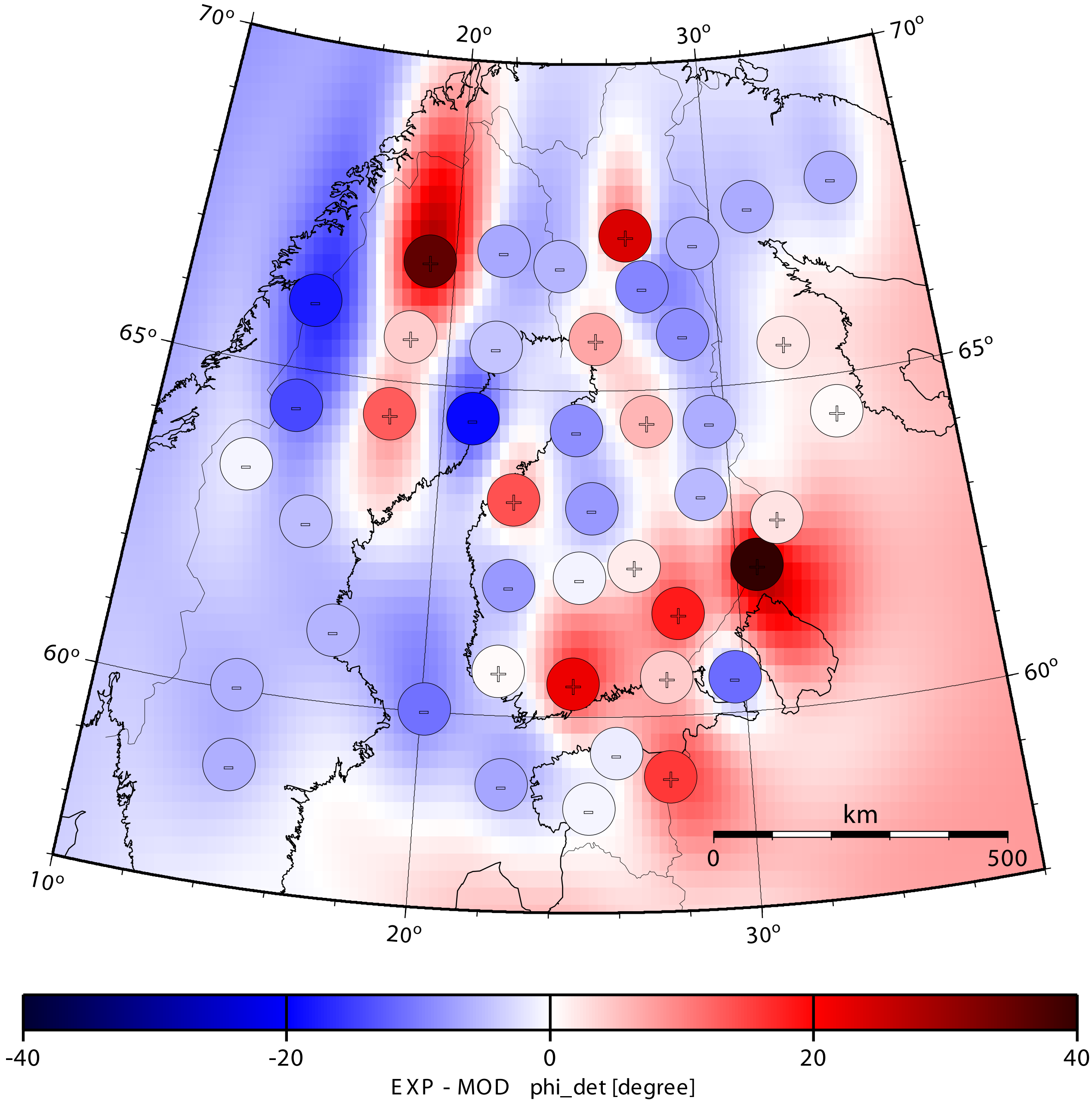

Figure 15: *Residuals in effective phase: Experiment minus model including an asthenosphere.*