3–D inversion of LOTEM data under strong boundary conditions

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Abstract

We present an inversion approach for long–offset transient electromagnetic (LOTEM) data using a solution scheme usually employed for a small number of model parameters, such as in 1–D inversions, in combination with a 3–D forward modeling algorithm. If an appropriate mathematical description is available, a least–squares inversion scheme can be used to estimate earth models composed by any set of parameters which simulate the underground. Subject to the condition of a limited number of parameters, the well–known Marquardt–Levenberg method provides us with a versatile scheme with mathematical robustness to estimate such parameters. The combination with a 3–D finite–difference code makes the approach efficient in the presence of a–priori information when inverting for 3–D underground structures. The shown example is an inversion of LOTEM data for a simple layered model based upon the surface topography of Mount Merapi (Indonesia).

Introduction

Algorithms for simulating responses of geologically realistic 3–D earth models are increasingly employed for the interpretation of geoelectric and electromagnetic measurements. However, when trying to fit observed data, the successive application of a 3–D forward modeling code either in a trial–and–error procedure or in a more sophisticated inversion approach is still limited by present computer capabilities. When using finite–difference or finite–element schemes in a modeling algorithm, a realistic representation of a geologically complex underground may require a large number of discrete grid cells to assign different values of physical properties. Hence the treatment of 3–D inversion problems usually involves a large number of unknowns which form the model parameter space. Robust inversion methods, such as the Singular Value Truncation technique [*Madden*, 1972] or the Marquardt–Levenberg method [*Marquardt*, 1963; *Levenberg*, 1944] have been applied successfully for the solution of small–scaled 1–D inversion problems. Yet with a large number of model unknowns the commonly used matrix inversion techniques, such as the Singular Value Decomposition, become less feasible.
Common to the majority of geophysical inverse problems is the non–uniqueness due to an insufficient covering of the target underground structure and the presence of data errors. It is common practice to reduce the non–uniqueness by restricting the complexity of the earth models [*Jupp and Vozoff*, 1975]. One approach is to impose additional conditions, such as smoothing constraints to the solution. For the one–dimensional inversion of TEM data *Farquharson and Oldenburg* [1993] justify this method by the argument that it avoids the risk of an overinterpretation in the case of a presence of high parameter contrasts, which may occur in a classical

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unconstrained least-squares solution. *DeGroot-Hedlin and Constable* [1990] argue similarly when presenting a 2–D inversion approach for magnetotelluric (MT) data by mentioning that smooth models reflect the true resolving power of the MT method. After all, *Newman and Alumbaugh* [1997] also mention the necessity of a stabilizing regularization term by a finite–difference approximation to the Laplacian operator to keep their 3–D inverse solution from being unstable and ill–posed. However, avoiding model features which are possibly not required by the data holds the risk of loosing some detail of a model by broadening true structure boundaries and dimensions. One might argue that the penalty for possibly realistic structures may be removed if adequate a–priori data is present. However, we believe that if only a distinct feature of the underground is of interest and physical properties and structure dimensions are already estimated from other methods or the underlying geology, an inversion approach for low–parameterized models seems more suitable. In order to benefit from a stable inversion scheme along with the flexibility of a 3–D modeling code, a combined approach is presented in this work.

Brief review: The Marquardt–Levenberg inversion scheme

As our inversion scheme is based upon the Marquardt–Levenberg method, the principles shall be briefly reviewed. For a more detailed treatment, the reader is referred to *Levenberg* [1944]; *Marquardt* [1963]; *Jupp and Vozoff* [1975]; *Lines and Treitel* [1984]. The principle of the method is to constrain the change of the model parameters $\mathbf{\Delta m}$ by bounding its squared values or energy by a finite quantity $\mathbf{m}_0^2$. This results in a damped least–squares solution. Inverting a set of observed data $\mathbf{y}$ for an earth model described by a parameter vector $\mathbf{m}$ basically involves the minimization of the cost functional [*Jennings and Osborne*, 1970; *Jupp and Vozoff*, 1975]

$$\mathrm{F}(\mathbf{\Delta m}, \beta) = \epsilon^{\mathrm{T}}\epsilon + \beta(\mathbf{\Delta m}^{\mathrm{T}}\mathbf{\Delta m} - \mathbf{\Delta m}_0^2) \tag{1}$$

where ϵ is called the error vector and represents the misfit between observed data and model response $\mathbf{f(m)}$

$$\epsilon = \mathbf{y} - \mathbf{f(m)}$$

For EM methods the function $\mathbf{f(m)}$ is non–linear. To use the techniques for linear inversion problems, it is usually represented by a first–order Taylor expansion around the initial model response $\mathbf{f}_0$

$$\mathbf{f(m)} = \mathbf{f}_0 + \mathbf{J\Delta m} + O(\mathbf{m}^2)$$

The matrix $\mathbf{J}$ therefore represents the partial derivatives of the predicted data with respect to the model parameters, also referred to as parameter sensitivity matrix or Jacobian. Basically a high value of the Lagrange multiplier β would enhance the influence of the second term on the right side of equation (1) at the expense of the error term $\epsilon^{\mathrm{T}}\epsilon$. This results in damping changes in the parameter vector $\mathbf{\Delta m}$ which also becomes obvious from the solution of minimizing equation (1) with respect to $\mathbf{\Delta m}$

$$\mathbf{\Delta m} = (\mathbf{J}^{\mathrm{T}}\mathbf{J} + \beta\mathbf{I})^{-1}\mathbf{J}^{\mathrm{T}}(\mathbf{y} - \mathbf{f}_0) \tag{2}$$

where $\mathbf{I}$ denotes the identity matrix. The robustness of the Marquardt–Levenberg approach has its reason in a combination of the method of steepest descent with the method of least–squares [*Lines and Treitel*, 1984].

Finite difference–algorithm and inversion scheme

For the example shown below the finite–difference code from *Druskin and Knizhnerman* [1988] was used for calculating the earth response for an electric dipole transmitter as is usually used in LOTEM surveys. The code is based on the spectral Lanczos decomposition method. It allows

a model parameterization using rectangular blocks which are independent from the underlying finite–difference grid. This is realized by inverse–interpolation of the physical properties of the consituents back into the mesh nodes [*Hördt and Müller*, 2000]. Thus one has the flexibility of specifying dimensions and boundaries of the model blocks without the necessity of a 3–D mesh design adapted to the shape of the assumed earth model. This capability provides a simple interface for handling the model block dimensions together with the inversion scheme without actually changing the modeling code. However, due to the general attenuation characteristics of EM fields the finite–difference grid has to be designed according to the electrical properties of the grid cells. *Hördt and Müller* [2000] note that the ratio between smallest and largest eigenvalue of the finite–difference matrix (grid condition number) has to be limited by a selective discretization. They mention the general rule, that the grid discretization should be fine in conductive regions and coarse in more resistive regions. Another difficulty may emerge from the required time range of the simulated data. To avoid numerical dispersion, which occurs whenever a finite grid is unable to simulate a high–frequency field [*Wang and Hohmann*, 1993], the spatial sampling rate must be sufficient. Yet, in order to limit the condition number at late times, a coarser grid is more accetable for later times. In the absence of a compromise between these two requirements, one may simulate early and late times using a fine and a coarse grid, respectively.
The inversion scheme presented here is the combination of a Marquardt–Levenberg method as a stable inversion scheme for a small number of model parameters with the described code. A great advantage of the Marquardt–Levenberg method is its robustness in the presence of data which is in the words of *Jackson* [1972] "inaccurate, insufficient and inconsistent". The combination offers the capability to invert for earth models with 3–D features, yet their complexity is limited to such an extent as is the number of model unknowns in a Marquardt–Levenberg method.

Example, inversion with topography

Approximating the surface topography of a survey area with a finite–difference scheme may require a high number of model blocks to account for rough surfaces. Figure 1 approximates the topography of the volcano Merapi [*Müller et al.*, 2002] which is located on the island of Java (Indonesia). The varying diameter of the vertical columns allows a refinement where the digital elevation model [*Gerstenecker et al.*, 1998] exhibits a rough surface. In addition to the terrain structure this scheme would also allow to model complex underground structures by assigning different physical properties to different sections of each column. As not illustrated in the figure, the air space above the surface is approximated by a highly–resistive column section extending from the surface upwards to an elevation reaching the actual vertical mesh boundary. Thus modeling a homogenous mountain only requires specifying the air–columns and assigning a background resistivity value to the remaining underground. Figure 2 shows two different responses of this mountain model in comparison with measured data. Curve 1 shows the transient of a vertical magnetic field measured during a long offset TEM survey in the summit area of Mount Merapi. The response of a homogeneous (500 Ωm) mountain is shown by curve 2. In fact this response differs marginally from that of a homogeneous halfspace with a flat surface. *Hördt and Müller* [2000] studied the effects of mountainous terrain on LOTEM data and found that the total effect of a mountain can be understood as a combination of two effects. The first is an early–time amplitude increase caused by moving the receiver away from the surface, and the second is a spatially antisymmetric early–time effect similar to that caused by a near–surface conductor.

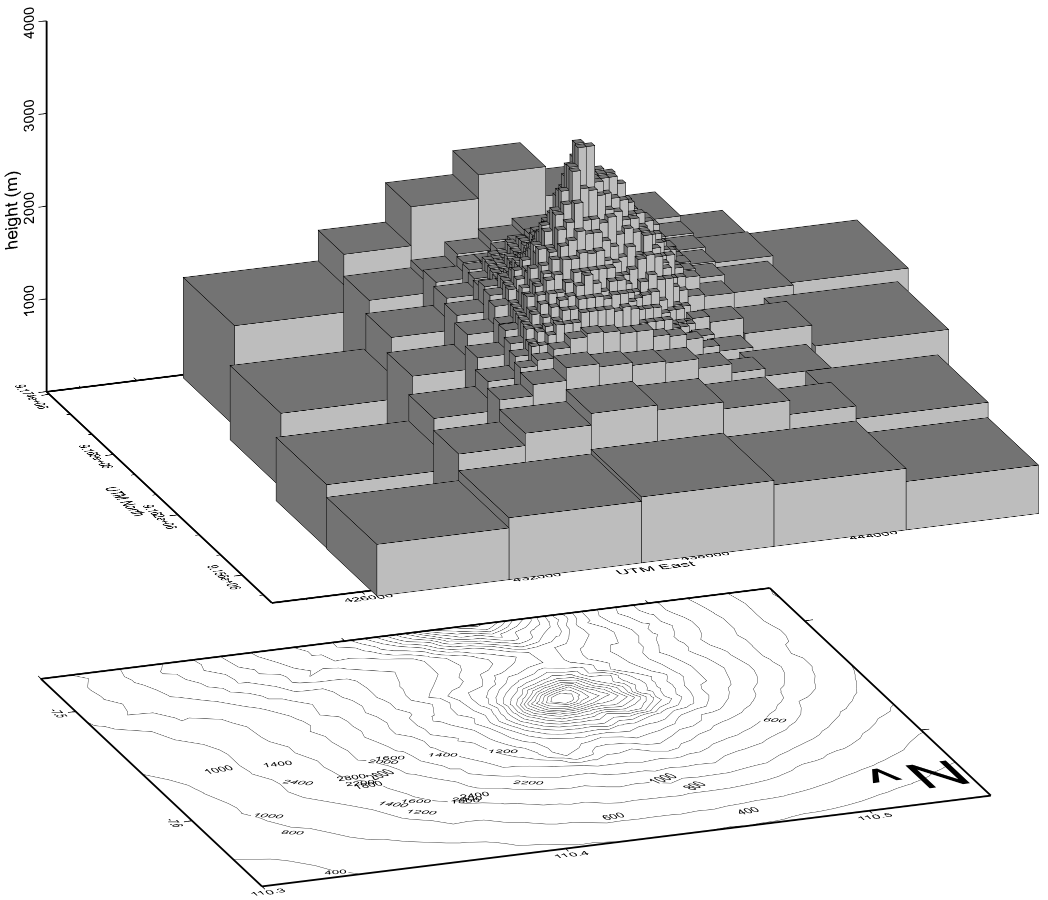

Figure 1: Modeling of the surface topography of Mount Merapi (Indonesia) with vertical columns.

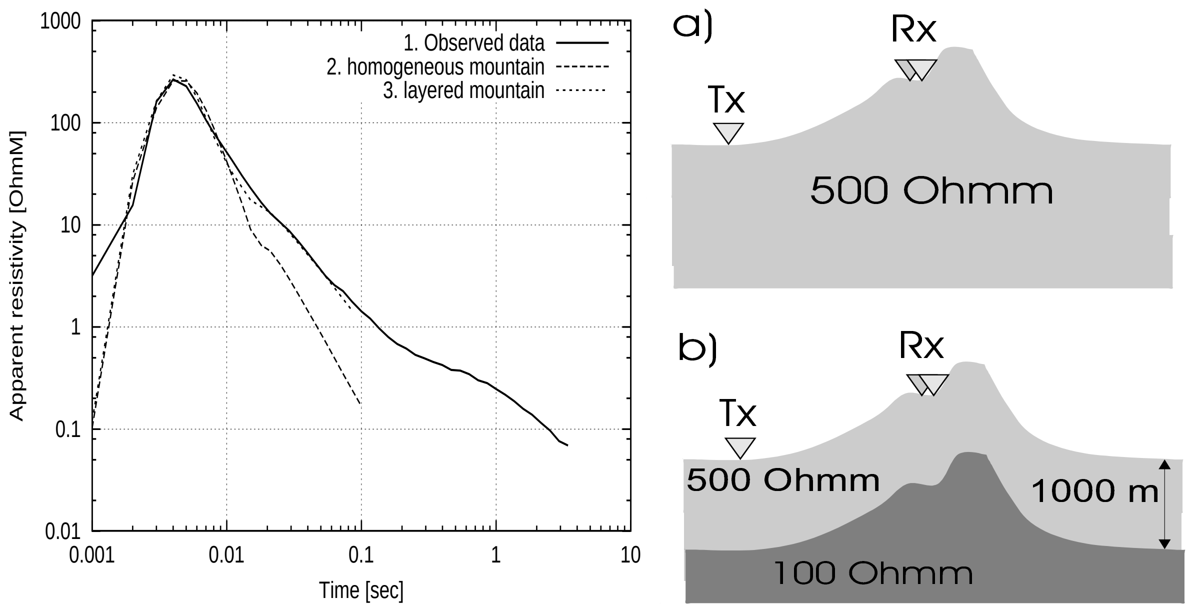

Figure 2: Comparison of measured vertical magnetic field transient with 3–D model responses from homogeneous (a) and layered mountain model (b).

As these effects are restricted to the early times, they would be not visible in the figure since the earliest observed value starts at 1 msec where, regarding the transmitter–receiver geometry for this data example, the late time range of the simulated curve begins. However, the example is intended to illustrate how the terrain structure serves as a constraining condition in an inversion for earth models with topography. Inverting for the homogenous mountain model would actually involve the variation of only one model parameter, i.e. the resistivity of the underground below the columns which form the free space above. Yet, the fast decay of the corresponding response (curve 2) after a time of 10 msec indicates the necessity of lower resistivities at larger depths to account for the relatively slow decay in the measured late times. The layered model b) in figure 2 is constructed by assigning the shape of the surface topography to the border of the two underground layers. Therefore, the distance between surface and top of the second layer is constant within each column of the whole block model. Hence, in addition to the two resistivity values, the thickness of the top layer is another varied parameter. In practice one lets the columns continue downwards to the lower mesh boundary and divides them vertically according to that thickness. The layered mountain could be viewed as an one–dimensional earth model which is somewhat deformed matching the topography. Thus, with only three parameters, an inversion can be performed with relatively low computational effort. As can be seen from the corresponding response (curve 3) the layered model achieves a better agreement with the original data than a homogeneous mountain.

Inversion results

Figures 3 and 4 show the results of inversions for layered mountain models. The data of a selected time window of two vertical magnetic field transients was inverted under the constraints that the shape of the layer boundary is fixed by the surface topography (see the schematic view in figure 3). Both data curves were measured in the summit area of Mount Merapi.
For the two–layered case the root mean square misfit between simulated and observed data converged relatively quickly after 7 iteration steps. The resulting 3–D model fits the measured transients in the chosen time range to a satisfying level as shown by figure 4. In agreement with 1–D results (not shown), one obtains a strong decrease of the resistivity below the first layer which has a thickness of approximately 2 km. Since flat 1–D earth models give very similar responses the shown time window of the data curve can be reproduced by both one–dimensional and three–dimensional (with surface topography) layered undergrounds. Note that, as mentioned above, the topography effects can be expected to be very small for this time range. Similar data misfits (figure 4) are achieved by a three–layered underground, allowing 5 model parameters to vary. The schematic view of the inversion result in figure 3 also shows a significant decrease of the resistivity with depth. This solution converged after 5 iterations. The full time range of one of the measured transients can be seen from the observed data curve in figure 2. Common to both inverted data curves is the significant decrease of decay at late times. The slope decreases in the late time section at the time around 0.2 sec which may suggest a very conductive structure in a large depth. However, a 1–D inversion result could not approximate this trend in the late time curve. For approximating the late times it may therefore be necessary to add 3–D structures to the composition of the model.
At this stage we emphasize the advantages of stability, as shown by the constant decrease of the r.m.s. errors, and the relatively low computational effort. The results shown were achieved in an average computation time of 2.5 hours on a 1 GHz processor machine. This time need can be further reduced by an easy to implement parallel approach. One has the alternative to distribute the calculation of the Jacobian matrix to several processors such that the forward modeling steps for the parameter perturbations are done simultaneously on different processors.

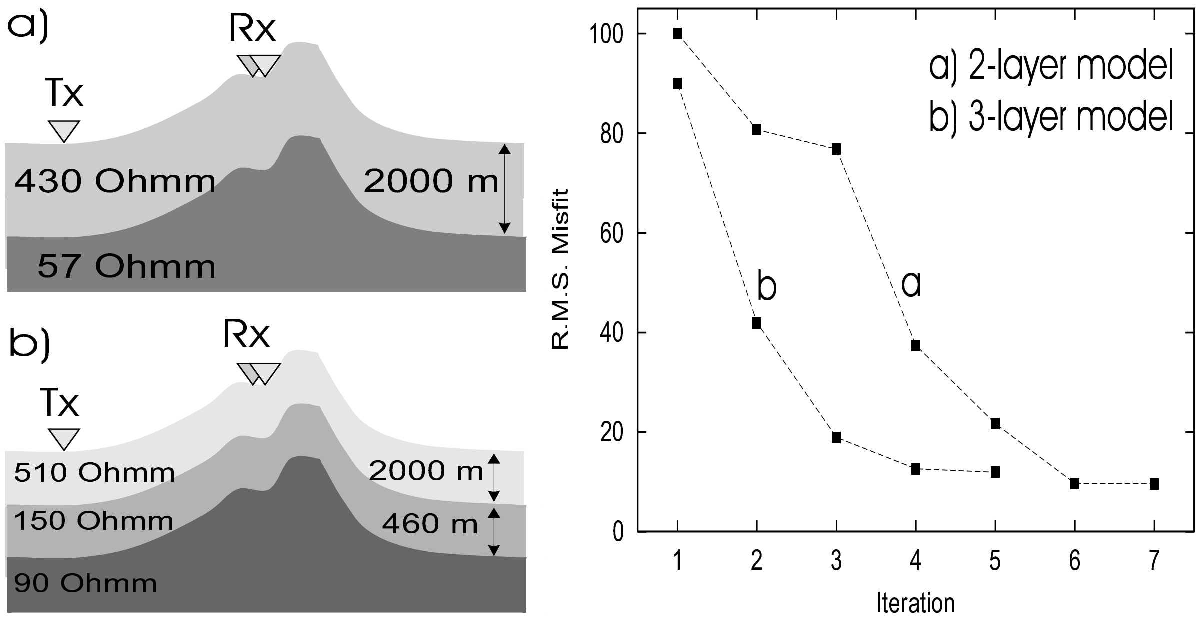

Figure 3: Left: Inversion results for layered earth models of Mount Merapi for a) two layers and b) three layers which are shaped according to the surface topography. Right: R.M.S. misfit as a function of the iteration number for both inversions.

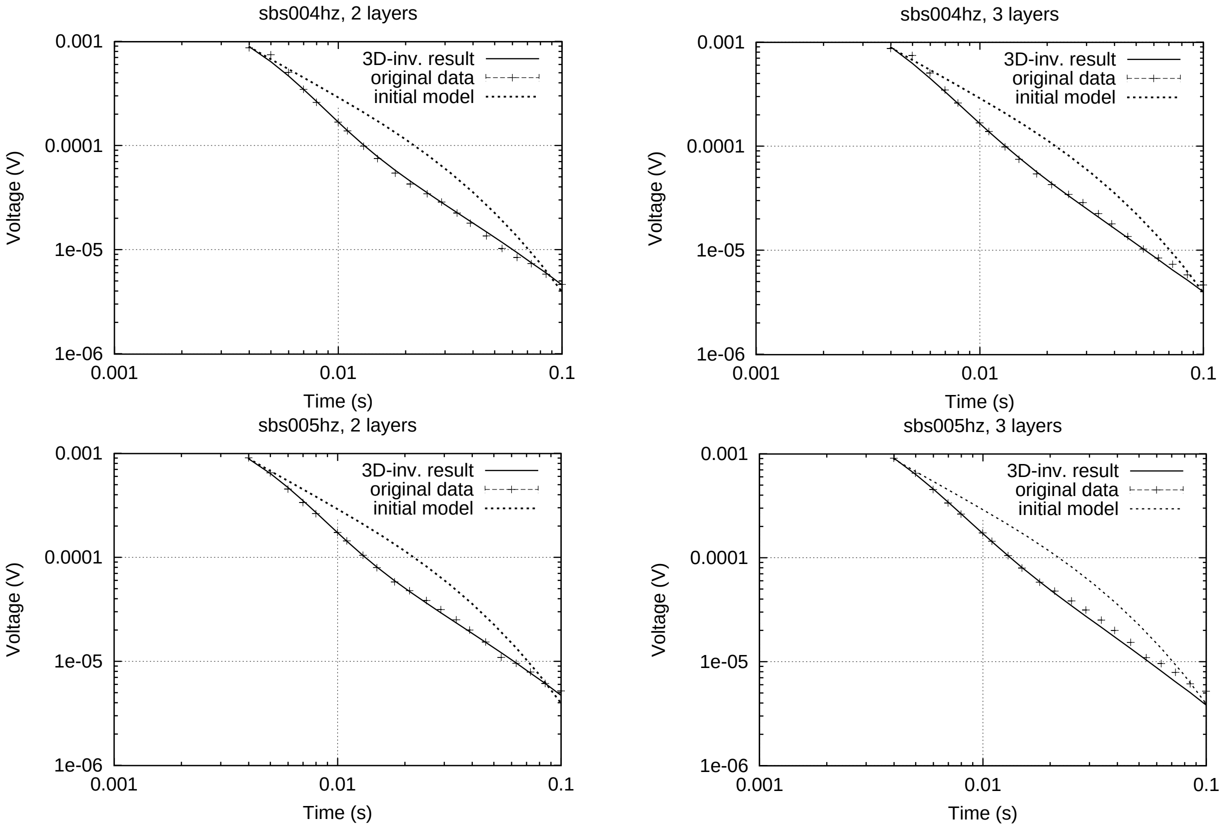

Figure 4: Data fit of the two measured transients for the inversion results from figure 3. Left: Two model parameters $(\varrho_1, \varrho_2, h_1)$, Right: Three model parameters $(\varrho_1, \varrho_2, \varrho_3, h_1, h_2)$

Conclusions

An inversion approach is presented combining the advantages of stability and robustness of the Marquardt–Levenberg inversion method with the flexibility of a 3–D forward modeling code. Although limited in terms of model parameterization the scheme is characterized by a relatively low computational effort. The computation times can be further reduced by a parallel approach for the calculation of the Jacobian.

For the inverted data examples shown an improvement for a data interpretation is achieved by including the topography of the survey area in the models. However, for the shown time windows layered models without surface topography (1–D) are also adequate for a data fit. In addition, a model of the type with layer boundaries matching the surface topography seems not sufficient to fit the late times of the shown transients. For future work it is planned to include additional 3–D structures in the mountain model. In a more detailed quantitative interpretation, topographic effects may become important. An example is a simulation of a low–resisistive structure below the summit area which is also proposed by the results of other geophysical measurements [*Maercklin et al.*, 2000].

The positive result at this stage is the successful integration of topography into the parameterization of the mountain model. If a 3–D model is constructed with a large number of rectangular blocks one has to impose constraining conditions on the totality of blocks to limit the number of degrees of freedom. The inversion example with a constraining condition subject to the underlying terrain shape shows that the Marquardt–Levenberg method yields stable results in combination with a 3–D forward code. The method may be efficient when inverting for models containing 3–D features if adequate a–priori information about the underground structure is present.

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